# Chapter 13
# Statistics
## Exercise 13.1

**Question** 1:

Find the mean deviation about the mean for the data.

4, 7, 8, 9, 10, 12, 13, 17

**Answer**:

The given data is.

4, 7, 8, 9, 10, 12, 13, 17

Mean of the data, $\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$

The deviations of the respective observations from the mean $\bar{x}$, i.e. $x_i - \bar{x}$, are.

−6, − 3, −2, −1, 0, 2, 3, 7

The absolute values of the deviations, i.e. $|x_i - \bar{x}|$ , are

6, 3, 2, 1, 0, 2, 3, 7

The required mean deviation about the mean is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{8} |x_i - \bar{x}|}{8} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$$

**Question** 2:

Find the mean deviation about the mean for the data

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

**Answer**:

The given data is

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Mean of the given data,

$$\bar{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$$

The deviations of the respective observations from the mean $\bar{x}$, i.e. $x_i - \bar{x}$, are

−12, 20, −2, −10, −8, 5, 13, −4, 4, −6

The absolute values of the deviations, i.e. $|x_i - \bar{x}|$ , are

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

The required mean deviation about the mean is

$$\begin{aligned} \text{M.D.}(\bar{x}) &= \frac{\sum_{i=1}^{10} |x_i - \bar{x}|}{10} \\ &= \frac{12+20+2+10+8+5+13+4+4+6}{10} \\ &= \frac{84}{10} \\ &= 8.4 \end{aligned}$$

**Question** 3:

Find the mean deviation about the median for the data.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

**Answer**:

The given data is

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Here, the numbers of observations are 12, which is even.

Arranging the data in ascending order, we obtain

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{Median, M} = \frac{\left(\frac{12}{2}\right)^{th} \text{observation} + \left(\frac{12}{2}+1\right)^{th} \text{observation}}{2}$$
$$= \frac{6^{th} \text{ observation} + 7^{th} \text{ observation}}{2}$$
$$= \frac{13+14}{2} = \frac{27}{2} = 13.5$$

The deviations of the respective observations from the median, i.e. $x_i - \text{M},$ are

−3.5, −2.5, −2.5, −1.5, −0.5, −0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The absolute values of the deviations, $|x_i - \text{M}|$ , are

3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The required mean deviation about the median is

$$\text{M.D.}(\text{M}) = \frac{\sum_{i=1}^{12} |x_i - \text{M}|}{12}$$
$$= \frac{3.5+2.5+2.5+1.5+0.5+0.5+0.5+2.5+2.5+3.5+3.5+4.5}{12}$$
$$= \frac{28}{12} = 2.33$$

**Question** 4:

Find the mean deviation about the median for the data

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

**Answer**:

The given data is

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Here, the number of observations is 10, which is even.

Arranging the data in ascending order, we obtain

36, 42, 45, 46, 46, 49, 51, 53, 60, 72

$$\text{Median M} = \frac{\left(\frac{10}{2}\right)^{th}\text{observation} + \left(\frac{10}{2}+1\right)^{th}\text{observation}}{2}$$
$$= \frac{5^{th}\text{observation} + 6^{th}\text{observation}}{2}$$
$$= \frac{46+49}{2} = \frac{95}{2} = 47.5$$

The deviations of the respective observations from the median, i.e. $x_i - \text{M},$ are

−11.5, −5.5, −2.5, −1.5, −1.5, 1.5, 3.5, 5.5, 12.5, 24.5

The absolute values of the deviations, $|x_i - \text{M}|$ , are

11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

Thus, the required mean deviation about the median is

$$\text{M.D.}(\text{M}) = \frac{\sum_{i=1}^{10}|x_i - \text{M}|}{10} = \frac{11.5+5.5+2.5+1.5+1.5+1.5+3.5+5.5+12.5+24.5}{10}$$
$$= \frac{70}{10} = 7$$

**Question** 5:

Find the mean deviation about the mean for the data.

| *xi* | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| *fi* | 7 | 4 | 6 | 3 | 5 |

**Answer**:

| ***xi*** | ***fi*** | ***fi xi*** | $\lvert x_i - \bar{x}\rvert$ | $f_i\lvert x_i - \bar{x}\rvert$ |
|---|---|---|---|---|
| 5 | 7 | 35 | 9 | 63 |
| 10 | 4 | 40 | 4 | 16 |
| 15 | 6 | 90 | 1 | 6 |
| 20 | 3 | 60 | 6 | 18 |
| 25 | 5 | 125 | 11 | 55 |
| | 25 | 350 | | 158 |

$$N = \sum_{i=1}^{5} f_i = 25$$

$$\sum_{i=1}^{5} f_i x_i = 350$$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

$$\therefore MD(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - \bar{x}| = \frac{1}{25} \times 158 = 6.32$$

**Question** 6:

Find the mean deviation about the mean for the data

| *xi* | 10 | 30 | 50 | 70 | 90 |
|---|---|---|---|---|---|
| *fi* | 4 | 24 | 28 | 16 | 8 |

**Answer**:

| ***xi*** | ***fi*** | ***fi xi*** | $\lvert x_i - \bar{x} \rvert$ | $f_i \lvert x_i - \bar{x} \rvert$ |
|---|---|---|---|---|
| 10 | 4 | 40 | 40 | 160 |
| 30 | 24 | 720 | 20 | 480 |
| 50 | 28 | 1400 | 0 | 0 |
| 70 | 16 | 1120 | 20 | 320 |
| 90 | 8 | 720 | 40 | 320 |
| | 80 | 4000 | | 1280 |

$$N = \sum_{i=1}^{5} f_i = 80, \quad \sum_{i=1}^{5} f_i x_i = 4000$$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{80} \times 4000 = 50$$

$$MD(\bar{x}) \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - \bar{x}| = \frac{1}{80} \times 1280 = 16$$

**Question** 7:

Find the mean deviation about the median for the data.

| ***xi*** | 5 | 7 | 9 | 10 | 12 | 15 |
|---|---|---|---|---|---|---|
| ***fi*** | 8 | 6 | 2 | 2 | 2 | 6 |

**Answer**:

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

| *xi* | *fi* | *c.f.* |
|---|---|---|
| 5 | 8 | 8 |
| 7 | 6 | 14 |
| 9 | 2 | 16 |
| 10 | 2 | 18 |
| 12 | 2 | 20 |
| 15 | 6 | 26 |

Here, N = 26, which is even.

Median is the mean of 13th and 14th observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.

$$\therefore \text{Median} = \frac{13^{th}\text{ observation} + 14^{th}\text{ observation}}{2} = \frac{7+7}{2} = 7$$

The absolute values of the deviations from median, i.e. $|x_i - \text{M}|$, are

| ***\|xi − M\|*** | 2 | 0 | 2 | 3 | 5 | 8 |
|---|---|---|---|---|---|---|
| ***fi*** | 8 | 6 | 2 | 2 | 2 | 6 |
| ***fi \|xi − M\|*** | 16 | 0 | 4 | 6 | 10 | 48 |

$\sum_{i=1}^{6} f_i = 26$ and $\sum_{i=1}^{6} f_i |x_i - \text{M}| = 84$

$$\text{M.D.}(\text{M}) = \frac{1}{\text{N}} \sum_{i=1}^{6} f_i |x_i - \text{M}| = \frac{1}{26} \times 84 = 3.23$$

**Question** 8:

Find the mean deviation about the median for the data

| ***xi*** | 15 | 21 | 27 | 30 | 35 |
|---|---|---|---|---|---|
| ***fi*** | 3 | 5 | 6 | 7 | 8 |

**Answer**:

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

| *xi* | *fi* | *c.f.* |
|---|---|---|
| 15 | 3 | 3 |

| | | |
|---|---|---|
| 21 | 5 | 8 |
| 27 | 6 | 14 |
| 30 | 7 | 21 |
| 35 | 8 | 29 |

Here, N = 29, which is odd.

$\therefore \text{Median} = \left(\frac{29+1}{2}\right)^{th}$ observation = $15^{th}$ observation

This observation lies in the cumulative frequency 21, for which the corresponding observation is 30.

∴ Median = 30

The absolute values of the deviations from median, i.e. $|x_i - \text{M}|$, are

| ***\|xi − M\|*** | 15 | 9 | 3 | 0 | 5 |
|---|---|---|---|---|---|
| ***fi*** | 3 | 5 | 6 | 7 | 8 |
| ***fi \|xi − M\|*** | 45 | 45 | 18 | 0 | 40 |

$\sum_{i=1}^{5} f_i = 29, \ \sum_{i=1}^{5} f_i |x_i - \text{M}| = 148$

∴ $\text{M.D.}(\text{M}) = \frac{1}{\text{N}} \sum_{i=1}^{5} f_i |x_i - \text{M}| = \frac{1}{29} \times 148 = 5.1$

**Question** 9:

Find the mean deviation about the mean for the data.

| Income per day | Number of persons |
|---|---|
| 0-100 | 4 |
| 100-200 | 8 |
| 200-300 | 9 |
| 300-400 | 10 |
| 400-500 | 7 |
| 500-600 | 5 |
| 600-700 | 4 |
| 700-800 | 3 |

**Answer**:

The following table is formed.

| Income per day | Number of persons *fi* | Mid-point *xi* | *fi xi* | $\lvert x_i - \bar{x} \rvert$ | $f_i \lvert x_i - \bar{x} \rvert$ |
|---|---|---|---|---|---|
| 0 – 100 | 4 | 50 | 200 | 308 | 1232 |
| 100 – 200 | 8 | 150 | 1200 | 208 | 1664 |
| 200 – 300 | 9 | 250 | 2250 | 108 | 972 |
| 300 – 400 | 10 | 350 | 3500 | 8 | 80 |
| 400 – 500 | 7 | 450 | 3150 | 92 | 644 |
| 500 – 600 | 5 | 550 | 2750 | 192 | 960 |
| 600 – 700 | 4 | 650 | 2600 | 292 | 1168 |
| 700 – 800 | 3 | 750 | 2250 | 392 | 1176 |
| | 50 | | 17900 | | 7896 |

Here, $N = \sum_{i=1}^{8} f_i = 50, \ \sum_{i=1}^{8} f_i x_i = 17900$

$$\therefore \bar{x} = \frac{1}{N} \sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

$$\text{M.D.}(\bar{x}) = \frac{1}{N} \sum_{i=1}^{8} f_i \lvert x_i - \bar{x} \rvert = \frac{1}{50} \times 7896 = 157.92$$

**Question** 10:

Find the mean deviation about the mean for the data

| Height in cms | Number of boys |
|---|---|
| 95-105 | 9 |
| 105-115 | 13 |
| 115-125 | 26 |
| 125-135 | 30 |
| 135-145 | 12 |
| 145-155 | 10 |

**Answer**:

The following table is formed.

| Height in cms | Number of boys *fi* | Mid-point *xi* | *fi xi* | $\lvert x_i - \bar{x} \rvert$ | $f_i \lvert x_i - \bar{x} \rvert$ |
|---|---|---|---|---|---|
| 95-105 | 9 | 100 | 900 | 25.3 | 227.7 |
| 105-115 | 13 | 110 | 1430 | 15.3 | 198.9 |
| 115-125 | 26 | 120 | 3120 | 5.3 | 137.8 |
| 125-135 | 30 | 130 | 3900 | 4.7 | 141 |
| 135-145 | 12 | 140 | 1680 | 14.7 | 176.4 |
| 145-155 | 10 | 150 | 1500 | 24.7 | 247 |

Here, $N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}| = \frac{1}{100} \times 1128.8 = 11.28$$

**Question** 11:

Find the mean deviation about median for the following data:

| Marks | Number of girls |
|---|---|
| 0-10 | 6 |
| 10-20 | 8 |
| 20-30 | 14 |
| 30-40 | 16 |
| 40-50 | 4 |
| 50-60 | 2 |

**Answer**:

The following table is formed.

| **Marks** | **Number of girls *fi*** | **Cumulative frequency (c.f.)** | **Mid-point *xi*** | ***\|xi* â€“ Med.\|** | ***fi \|xi* â€“ Med.\|** |
|---|---|---|---|---|---|
| 0-10 | 6 | 6 | 5 | 22.85 | 137.1 |
| 10-20 | 8 | 14 | 15 | 12.85 | 102.8 |
| 20-30 | 14 | 28 | 25 | 2.85 | 39.9 |
| 30-40 | 16 | 44 | 35 | 7.15 | 114.4 |
| 40-50 | 4 | 48 | 45 | 17.15 | 68.6 |
| 50-60 | 2 | 50 | 55 | 27.15 | 54.3 |
| | 50 | | | | 517.1 |

The class interval containing the $\left(\frac{N}{2}\right)^{th}$ or 25th item is 20 â€“ 30.

Therefore, 20 â€“ 30 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, *l* = 20, C = 14, *f* = 14, *h* = 10, and N = 50

âˆ´ Median = $20+\frac{25-14}{14}\times 10 = 20+\frac{110}{14} = 20+7.85 = 27.85$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(\text{M}) = \frac{1}{\text{N}}\sum_{i=1}^{6} f_i|x_i - \text{M}| = \frac{1}{50}\times 517.1 = 10.34$$

**Question** 12:

Calculate the mean deviation about median age for the age distribution of 100 persons given below:

| Age | Number |
|---|---|
| 16-20 | 5 |
| 21-25 | 6 |
| 26-30 | 12 |
| 31-35 | 14 |
| 36-40 | 26 |
| 41-45 | 12 |
| 46-50 | 16 |
| 51-55 | 9 |

**Answer**:

The given data is not continuous. Therefore, it has to be converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

The table is formed as follows.

| **Age** | **Number *fi*** | **Cumulative frequency (c.f.)** | **Mid-point *xi*** | **\|*xi* – Med.\|** | ***fi* \|*xi* – Med.\|** |
|---|---|---|---|---|---|
| 15.5-20.5 | 5 | 5 | 18 | 20 | 100 |
| 20.5-25.5 | 6 | 11 | 23 | 15 | 90 |
| 25.5-30.5 | 12 | 23 | 28 | 10 | 120 |
| 30.5-35.5 | 14 | 37 | 33 | 5 | 70 |
| 35.5-40.5 | 26 | 63 | 38 | 0 | 0 |
| 40.5-45.5 | 12 | 75 | 43 | 5 | 60 |
| 45.5-50.5 | 16 | 91 | 48 | 10 | 160 |
| 50.5-55.5 | 9 | 100 | 53 | 15 | 135 |
| | 100 | | | | 735 |

The class interval containing the $\frac{\text{N}^{th}}{2}$ or 50th item is 35.5 – 40.5.

Therefore, 35.5 – 40.5 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, $l$ = 35.5, C = 37, $f$ = 26, $h$ = 5, and N = 100

$$\therefore \text{Median} = 35.5 + \frac{50 - 37}{26} \times 5 \quad = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{8} f_i |x_i - M| = \frac{1}{100} \times 735 = 7.35$$

# Exercise 13.2

**Question** 1:

Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12

**Answer**:

6, 7, 10, 12, 13, 4, 8, 12

Mean, $\bar{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

The following table is obtained.

| $x_i$ | $(x_i - \bar{x})$ | $(x_i - \bar{x})^2$ |
|---|---|---|
| 6 | −3 | 9 |
| 7 | −2 | 4 |
| 10 | −1 | 1 |
| 12 | 3 | 9 |
| 13 | 4 | 16 |
| 4 | −5 | 25 |
| 8 | −1 | 1 |
| 12 | 3 | 9 |

| | | |
|---|---|---|
| | | 74 |

$$\text{Variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \frac{1}{8} \times 74 = 9.25$$

**Question** 2:

Find the mean and variance for the first *n* natural numbers

**Answer**:

The mean of first *n* natural numbers is calculated as follows.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\text{Variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[x_i - \left(\frac{n+1}{2}\right)\right]^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \frac{1}{n}\sum_{i=1}^{n}2\left(\frac{n+1}{2}\right)x_i + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{n}\right)\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}$$

**Question** 3:

Find the mean and variance for the first 10 multiples of 3

**Answer**:

The first 10 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here, number of observations, $n$ = 10

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5$$

The following table is obtained.

| ***xi*** | $(x_i - \bar{x})$ | $(x_i - \bar{x})^2$ |
|---|---|---|
| 3 | −13.5 | 182.25 |
| 6 | −10.5 | 110.25 |
| 9 | −7.5 | 56.25 |
| 12 | −4.5 | 20.25 |
| 15 | −1.5 | 2.25 |
| 18 | 1.5 | 2.25 |
| 21 | 4.5 | 20.25 |
| 24 | 7.5 | 56.25 |
| 27 | 10.5 | 110.25 |
| 30 | 13.5 | 182.25 |
| | | 742.5 |

$$\text{Variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \frac{1}{10} \times 742.5 = 74.25$$

**Question** 4:

Find the mean and variance for the data

| *xi* | 6 | 10 | 14 | 18 | 24 | 28 | 30 |
|---|---|---|---|---|---|---|---|
| *f i* | 2 | 4 | 7 | 12 | 8 | 4 | 3 |

**Answer**:

The data is obtained in tabular form as follows.

| ***xi*** | ***f i*** | ***fixi*** | $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ | $f_i(x_i - \bar{x})^2$ |
|---|---|---|---|---|---|
| 6 | 2 | 12 | −13 | 169 | 338 |
| 10 | 4 | 40 | −9 | 81 | 324 |
| 14 | 7 | 98 | −5 | 25 | 175 |
| 18 | 12 | 216 | −1 | 1 | 12 |
| 24 | 8 | 192 | 5 | 25 | 200 |
| 28 | 4 | 112 | 9 | 81 | 324 |

| | | | | | |
|---|---|---|---|---|---|
| 30 | 3 | 90 | 11 | 121 | 363 |
| | 40 | 760 | | | 1736 |

Here, N = 40, $\sum_{i=1}^{7} f_i x_i = 760$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{760}{40} = 19$$

$$\text{Variance} = (\sigma^2) = \frac{1}{N}\sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

**Question** 5:

Find the mean and variance for the data

| *xi* | 92 | 93 | 97 | 98 | 102 | 104 | 109 |
|---|---|---|---|---|---|---|---|
| *f i* | 3 | 2 | 3 | 2 | 6 | 3 | 3 |

**Answer**:

The data is obtained in tabular form as follows.

| ***xi*** | ***f i*** | ***fixi*** | $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ | $f_i (x_i - \bar{x})^2$ |
|---|---|---|---|---|---|
| 92 | 3 | 276 | −8 | 64 | 192 |
| 93 | 2 | 186 | −7 | 49 | 98 |
| 97 | 3 | 291 | −3 | 9 | 27 |
| 98 | 2 | 196 | −2 | 4 | 8 |
| 102 | 6 | 612 | 2 | 4 | 24 |
| 104 | 3 | 312 | 4 | 16 | 48 |
| 109 | 3 | 327 | 9 | 81 | 243 |
| | 22 | 2200 | | | 640 |

Here, N = 22, $\sum_{i=1}^{7} f_i x_i = 2200$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance}(\sigma^2) = \frac{1}{N}\sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$$

**Question** 6:

Find the mean and standard deviation using short-cut method.

| *xi* | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 |
|---|---|---|---|---|---|---|---|---|---|
| *fi* | 2 | 1 | 12 | 29 | 25 | 12 | 10 | 4 | 5 |

**Answer**:

The data is obtained in tabular form as follows.

| ***xi*** | ***fi*** | $f_i = \frac{x_i - 64}{1}$ | ***yi2*** | ***fiyi*** | ***fiyi2*** |
|---|---|---|---|---|---|
| 60 | 2 | −4 | 16 | −8 | 32 |
| 61 | 1 | −3 | 9 | −3 | 9 |
| 62 | 12 | −2 | 4 | −24 | 48 |
| 63 | 29 | −1 | 1 | −29 | 29 |
| 64 | 25 | 0 | 0 | 0 | 0 |
| 65 | 12 | 1 | 1 | 12 | 12 |
| 66 | 10 | 2 | 4 | 20 | 40 |
| 67 | 4 | 3 | 9 | 12 | 36 |
| 68 | 5 | 4 | 16 | 20 | 80 |
| | 100 | 220 | | 0 | 286 |

Mean, $\bar{x} = A\frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$

$$\text{Variance}, \sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$
$$= \frac{1}{100^2}[100 \times 286 - 0]$$
$$= 2.86$$
$$\therefore \text{Standard deviation}(\sigma) = \sqrt{2.86} = 1.69$$

**Question 7:**

Find the mean and variance for the following frequency distribution.

| Classes | 0-30 | 30-60 | 60-90 | 90-120 | 120-150 | 150-180 | 180-210 |
|---|---|---|---|---|---|---|---|
| Frequencies | 2 | 3 | 5 | 10 | 3 | 5 | 2 |

**Answer**:

| **Class** | **Frequency *fi*** | **Mid-point *xi*** | $y_i = \frac{x_i - 105}{30}$ | ***yi2*** | ***fiyi*** | ***fiyi2*** |
|---|---|---|---|---|---|---|
| 0-30 | 2 | 15 | −3 | 9 | −6 | 18 |
| 30-60 | 3 | 45 | −2 | 4 | −6 | 12 |
| 60-90 | 5 | 75 | −1 | 1 | −5 | 5 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 90-120 | 10 | 105 | 0 | 0 | 0 | 0 |
| 120-150 | 3 | 135 | 1 | 1 | 3 | 3 |
| 150-180 | 5 | 165 | 2 | 4 | 10 | 20 |
| 180-210 | 2 | 195 | 3 | 9 | 6 | 18 |
| | 30 | | | | 2 | 76 |

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$
$$= \frac{(30)^2}{(30)^2}\left[30 \times 76 - (2)^2\right]$$
$$= 2280 - 4$$
$$= 2276$$

**Question** 8:

Find the mean and variance for the following frequency distribution.

| Classes | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| Frequencies | 5 | 8 | 15 | 16 | 6 |

**Answer**:

| **Class** | **Frequency** ***fi*** | **Mid-point *xi*** | $y_i = \frac{x_i - 25}{10}$ | ***yi2*** | ***fiyi*** | ***fiyi2*** |
|---|---|---|---|---|---|---|
| 0-10 | 5 | 5 | −2 | 4 | −10 | 20 |
| 10-20 | 8 | 15 | −1 | 1 | −8 | 8 |
| 20-30 | 15 | 25 | 0 | 0 | 0 | 0 |
| 30-40 | 16 | 35 | 1 | 1 | 16 | 16 |
| 40-50 | 6 | 45 | 2 | 4 | 12 | 24 |
| | 50 | | | | 10 | 68 |

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^{\,2} - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}\left[50 \times 68 - (10)^2\right]$$

$$= \frac{1}{25}[3400 - 100] = \frac{3300}{25}$$

$$= 132$$

**Question** 9:

Find the mean, variance and standard deviation using short-cut method

| Height in cms | No. of children |
|---|---|
| 70-75 | 3 |
| 75-80 | 4 |
| 80-85 | 7 |
| 85-90 | 7 |
| 90-95 | 15 |
| 95-100 | 9 |
| 100-105 | 6 |
| 105-110 | 6 |
| 110-115 | 3 |

**Answer**:

| **Class Interval** | **Frequency *fi*** | **Mid-point *xi*** | $y_i = \frac{x_i - 92.5}{5}$ | ***yi2*** | ***fiyi*** | ***fiyi2*** |
|---|---|---|---|---|---|---|
| 70-75 | 3 | 72.5 | −4 | 16 | −12 | 48 |
| 75-80 | 4 | 77.5 | −3 | 9 | −12 | 36 |
| 80-85 | 7 | 82.5 | −2 | 4 | −14 | 28 |
| 85-90 | 7 | 87.5 | −1 | 1 | −7 | 7 |
| 90-95 | 15 | 92.5 | 0 | 0 | 0 | 0 |
| 95-100 | 9 | 97.5 | 1 | 1 | 9 | 9 |
| 100-105 | 6 | 102.5 | 2 | 4 | 12 | 24 |
| 105-110 | 6 | 107.5 | 3 | 9 | 18 | 54 |
| 110-115 | 3 | 112.5 | 4 | 16 | 12 | 48 |
| | 60 | | | | 6 | 254 |

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$

$$= \frac{(5)^2}{(60)^2}\left[60 \times 254 - (6)^2\right]$$

$$= \frac{25}{3600}(15204) = 105.58$$

$$\therefore \text{Standard deviation}(\sigma) = \sqrt{105.58} = 10.27$$

**Question** 10:

The diameters of circles (in mm) drawn in a design are given below:

**Answer**:

| Diameters | No. of children |
|---|---|
| 33-36 | 15 |
| 37-40 | 17 |
| 41-44 | 21 |
| 45-48 | 22 |
| 49-52 | 25 |

| **Class Interval** | **Frequency *fi*** | **Mid-point *xi*** | $y_i = \frac{x_i - 42.5}{4}$ | ***fi*2** | ***fiyi*** | ***fiyi*2** |
|---|---|---|---|---|---|---|
| 32.5-36.5 | 15 | 34.5 | −2 | 4 | −30 | 60 |
| 36.5-40.5 | 17 | 38.5 | −1 | 1 | −17 | 17 |
| 40.5-44.5 | 21 | 42.5 | 0 | 0 | 0 | 0 |
| 44.5-48.5 | 22 | 46.5 | 1 | 1 | 22 | 22 |
| 48.5-52.5 | 25 | 50.5 | 2 | 4 | 50 | 100 |
| | 100 | | | | 25 | 199 |

Here, N = 100, *h* = 4

Let the assumed mean, A, be 42.5.

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{16}{10000}\left[100 \times 199 - (25)^2\right]$$

$$= \frac{16}{10000}[19900 - 625]$$

$$= \frac{16}{10000} \times 19275$$

$$= 30.84$$

$$\therefore \text{Standard deviation}(\sigma) = 5.55$$

# Miscellaneous Exercise

**Question** 1:

The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

**Answer**:

Let the remaining two observations be $x$ and $y$.

Therefore, the observations are 6, 7, 10, 12, 12, 13, $x$, $y$.

$$\text{Mean, } \bar{x} = \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$\Rightarrow x + y = 12 \quad \ldots(1)$$

$$\text{Variance} = 9.25 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

$$9.25 = \frac{1}{8}\left[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2 \times 9(x+y) + 2 \times (9)^2\right]$$

$$9.25 = \frac{1}{8}\left[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18(12) + 162\right]$$

$$\ldots[\text{Using } (1)]$$

$$9.25 = \frac{1}{8}\left[48 + x^2 + y^2 - 216 + 162\right]$$

$$9.25 = \frac{1}{8}\left[x^2 + y^2 - 6\right]$$

$$\Rightarrow x^2 + y^2 = 80 \quad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 144$ …(3)

From (2) and (3), we obtain

$2xy = 64$ … (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 80 - 64 = 16$

$\Rightarrow x - y = \pm 4$ … (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 4$, when $x - y = 4$

$x = 4$ and $y = 8$, when $x - y = -4$

Thus, the remaining observations are 4 and 8.

**Question** 2:

The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12 and 14. Find the remaining two observations.

**Answer**:

Let the remaining two observations be $x$ and $y$.

The observations are 2, 4, 10, 12, 14, $x$, $y$.

$$\text{Mean, } \bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow 56 = 42 + x + y$$

$$\Rightarrow x + y = 14 \quad \ldots(1)$$

$$\text{Variance} = 16 = \frac{1}{n}\sum_{i=1}^{7}\left(x_i - \bar{x}\right)^2$$

$$16 = \frac{1}{7}\left[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2\times 8(x+y) + 2\times(8)^2\right]$$

$$16 = \frac{1}{7}\left[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)\right]$$

$\ldots$[Using (1)]

$$16 = \frac{1}{7}\left[108 + x^2 + y^2 - 224 + 128\right]$$

$$16 = \frac{1}{7}\left[12 + x^2 + y^2\right]$$

$$\Rightarrow x^2 + y^2 = 112 - 12 = 100$$

$$x^2 + y^2 = 100 \quad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ … (3)

From (2) and (3), we obtain

$2xy = 196 - 100$

$\Rightarrow 2xy = 96$ … (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 100 - 96$

$\Rightarrow (x - y)^2 = 4$

$\Rightarrow x - y = \pm 2$ … (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 6$ when $x - y = 2$

$x = 6$ and $y = 8$ when $x - y = -2$

Thus, the remaining observations are 6 and 8.

**Question** 3:

The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

**Answer**:

Let the observations be $x_1$, $x_2$, $x_3$, $x_4$, $x_5$, and $x_6$.

It is given that mean is 8 and standard deviation is 4.

$$\text{Mean}, \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8 \qquad \ldots(1)$$

If each observation is multiplied by 3 and the resulting observations are $y_i$, then

$$y_i = 3x_i \text{ i.e., } x_i = \frac{1}{3}y_i, \text{ for } i = 1 \text{ to } 6$$

$$\therefore \text{New mean, } \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$$= 3 \times 8 \qquad \ldots[\text{Using (1)}]$$

$$= 24$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i - \bar{x})^2}$$

$$\therefore (4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$$

$$\sum_{i=1}^{6}(x_i - \bar{x})^2 = 96 \qquad \ldots(2)$$

From (1) and (2), it can be observed that,

$$\bar{y} = 3\bar{x}$$

$$\bar{x} = \frac{1}{3}\bar{y}$$

Substituting the values of $x_i$ and $\bar{x}$ in (2), we obtain

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}\bar{y}\right)^2 = 96$$

$$\Rightarrow \sum_{i=1}^{6}(y_i - \bar{y})^2 = 864$$

Therefore, variance of new observations = $\left(\frac{1}{6}\times 864\right)=144$

Hence, the standard deviation of new observations is $\sqrt{144}=12$

**Question** 4:

Given that $\bar{x}$ is the mean and $\sigma^2$ is the variance of $n$ observations $x_1, x_2 \ldots xn$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3 \ldots axn$ are $\bar{ax}$ and $a^2\sigma^2$, respectively ($a \neq 0$).

**Answer**:

The given $n$ observations are $x_1, x_2 \ldots xn$.

Mean = $\bar{x}$

Variance = $\sigma^2$

$$\therefore \sigma^2=\frac{1}{n}\sum_{i=1}^{n} y_i\left(x_i-\bar{x}\right)^2 \quad \ldots(1)$$

If each observation is multiplied by $a$ and the new observations are $yi$, then

$$y_i=ax_i \text{ i.e., } x_i=\frac{1}{a}y_i$$

$$\therefore \bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i=\frac{1}{n}\sum_{i=1}^{n} ax_i=\frac{a}{n}\sum_{i=1}^{n} x_i=a\bar{x} \qquad \left(\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

Therefore, mean of the observations, $ax_1, ax_2 \ldots axn$, is $\bar{ax}$.

Substituting the values of $xi$and $\bar{x}$ in (1), we obtain

$$\sigma^2=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_i-\frac{1}{a}\bar{y}\right)^2$$

$$\Rightarrow a^2\sigma^2=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2$$

Thus, the variance of the observations, $ax_1, ax_2 \ldots axn$, is $a^2\sigma^2$

**Question** 5:

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculat the correct mean and standard deviation in each of the following cases:

**(i)** If wrong item is omitted.

**(ii)** If it is replaced by 12.

**Answer**:

**(i)** Number of observations ($n$) = 20

Incorrect mean = 10

Incorrect standard deviation = 2

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$$
$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$
$$\Rightarrow \sum_{i=1}^{20} x_i = 200$$

That is, incorrect sum of observations = 200

Correct sum of observations = 200 − 8 = 192

∴ Correct mean $= \frac{\text{Correct sum}}{19} = \frac{192}{19} = 10.1$

Standard deviation, σ = 1n∑i=1nxi2 − 1n2∑i=1nxi2⇒2 = 1n∑i=1nxi2 − 1n∑i=1nxi2

⇒2 = 1n∑i=1nxi2 − x¯2 as, 1n∑i=1nx = x ⇒2 = 120×Incorrect∑i=1nxi2 − 102

⇒4 = 120×Incorrect∑i=1nxi2 − 100⇒120×Incorrect∑i=1nxi2 = 104

⇒Incorrect∑i=1nxi2 = 2080Now, correct∑i=1nxi2 = Incorrect∑i=1nxi2 − 82

⇒correct∑i=1nxi2 = 2080 − 64 = 2016

∴ correct Standard Deviation = 1ncorrect∑i=1nxi2 − correct mean2

⇒correct Standard Deviation = 119×2016 − 192192

⇒correct Standard Deviation = 201619-192192

⇒correct Standard Deviation = 144019 = 121019

⇒correct Standard Deviation = 12 × 3.16219 = 1.997**(ii)** When 8 is replaced by 12,

Incorrect sum of observations = 200

∴ Correct sum of observations = 200 − 8 + 12 = 204

$$\therefore \text{Correct mean} = \frac{\text{Correct sum}}{20} = \frac{204}{20} = 10.2$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2 + (12)^2$$

$$= 2080 - 64 + 144$$

$$= 2160$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct}\sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2160}{20} - (10.2)^2}$$

$$= \sqrt{108 - 104.04}$$

$$= \sqrt{3.96}$$

$$= 1.98$$

**Question** 6:

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

**Answer**:

Number of observations (*n*) = 100

Incorrect mean $\left(\bar{x}\right) = 20$

Incorrect standard deviation $(\sigma)=3$

$$\Rightarrow 20=\frac{1}{100}\sum_{i=1}^{100}x_i$$

$$\Rightarrow \sum_{i=1}^{100}x_i=20\times100=2000$$

∴ Incorrect sum of observations = 2000

⇒ Correct sum of observations = 2000 − 21 − 21 − 18 = 2000 − 60 = 1940

$$\therefore \text{Correct mean}=\frac{\text{Correct sum}}{100-3}=\frac{1940}{97}=20$$

$$\text{Standard deviation}(\sigma)=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i-\frac{1}{n^2}\left(\sum_{i=1}^{n}x_i\right)^2}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-(\bar{x})^2}$$

$$\Rightarrow 3=\sqrt{\frac{1}{100}\times\text{Incorrect}\sum x_i^2-(20)^2}$$

$$\Rightarrow \text{Incorrect}\sum x_i^2=100(9+400)=40900$$

$$\text{Correct}\sum_{i=1}^{n}x_i^2=\text{Incorrect}\sum_{i=1}^{n}x_i^2-(21)^2-(21)^2-(18)^2$$

$$=40900-441-441-324$$

$$=39694$$

$$\therefore \text{Correct standard deviation}=\sqrt{\frac{\text{Correct}\sum x_i^2}{n}-(\text{Correct mean})^2}$$

$$=\sqrt{\frac{39694}{97}-(20)^2}$$

$$=\sqrt{409.216-400}$$

$$=\sqrt{9.216}$$

$$=3.036$$