Chapter 14

Probability

Exercise 14.1

Question 1:

A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Answer:

When a die is rolled, the sample space is given by

S = {1, 2, 3, 4, 5, 6}

Accordingly, E = {4} and F = {2, 4, 6}

It is observed that $E \cap F = \{4\} \neq \Phi$

Therefore, E and F are not mutually exclusive events.

Question 2:

A die is thrown. Describe the following events:

(i) A: a number less than 7 (ii) B: a number greater than 7 (iii) C: a multiple of 3

(iv) D: a number less than 4 (v) E: an even number greater than 4 (vi) F: a number not less than 3

Also find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, E \cap F', F'$

Answer:

When a die is thrown, the sample space is given by S = {1, 2, 3, 4, 5, 6}.

Accordingly:

(i) A = {1, 2, 3, 4, 5, 6}

(ii) B = Φ

(iii) C = {3, 6}

(iv) D = {1, 2, 3}

(v) E = {6}

(vi) F = {3, 4, 5, 6}

A ∪ B = {1, 2, 3, 4, 5, 6}, A ∩ B = Φ

B ∪ C = {3, 6}, E ∩ F = {6}

D ∩ E =Φ, A − C = {1, 2, 4, 5}

D − E = {1, 2, 3}, $F' = \{1,2\}$, $E \cap F' = \phi$

Question 3:

An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8, B: 2 occurs on either die

C: The sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Answer:

When a pair of dice is rolled, the sample space is given by

$$S = \{(x, y) : x, y = 1,2,3,4,5,6\}$$

$$= \left\{\begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix}\right\}$$

Accordingly,

$$A = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$$

$$B = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(1,2),(3,2),(4,2),(5,2),(6,2)\}$$

$$C = \{(3,6),(4,5),(5,4),(6,3),(6,6)\}$$

It is observed that

A ∩ B =Φ

$B \cap C = \Phi$

$C \cap A = \{(3,6),(4,5),(5,4),(6,3),(6,6)\} \neq \phi$

Hence, events A and B and events B and C are mutually exclusive.

Question 4:

Three coins are tossed once. Let A denote the event 'three heads show", B denote the event "two heads and one tail show". C denote the event "three tails show" and D denote the event 'a head shows on the first coin". Which events are

(i) mutually exclusive? (ii) simple? (iii) compound?

Answer:

When three coins are tossed, the sample space is given by

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Accordingly,

A = {HHH}

B = {HHT, HTH, THH}

C = {TTT}

D = {HHH, HHT, HTH, HTT}

We now observe that

$A \cap B = \Phi$, $A \cap C = \Phi$, $A \cap D = \{HHH\} \neq \Phi$

$B \cap C = \Phi$, $B \cap D = \{HHT, \{HTH\} \neq \Phi$

$C \cap D = \Phi$

(i) Event A and B; event A and C; event B and C; and event C and D are all mutually exclusive.

(ii) If an event has only one sample point of a sample space, it is called a simple event. Thus, A and C are simple events.

(iii) If an event has more than one sample point of a sample space, it is called a compound event. Thus, B and D are compound events.

Question 5:

Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii) Two events, which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Answer:

When three coins are tossed, the sample space is given by

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(i) Two events that are mutually exclusive can be

A: getting no heads and B: getting no tails

This is because sets A = {TTT} and B = {HHH} are disjoint.

(ii) Three events that are mutually exclusive and exhaustive can be

A: getting no heads

B: getting exactly one head

C: getting at least two heads

i.e.,

A = {TTT}

B = {HTT, THT, TTH}

C = {HHH, HHT, HTH, THH}

This is because $A \cap B = B \cap C = C \cap A = \Phi$ and $A \cup B \cup C = S$

(iii) Two events that are not mutually exclusive can be

A: getting three heads

B: getting at least 2 heads

i.e.,

A = {HHH}

B = {HHH, HHT, HTH, THH}

This is because A ∩ B = {HHH} ≠ Φ

(iv) Two events which are mutually exclusive but not exhaustive can be

A: getting exactly one head

B: getting exactly one tail

That is

A = {HTT, THT, TTH}

B = {HHT, HTH, THH}

It is because, A ∩ B =Φ, but A ∪ B ≠ S

(v) Three events that are mutually exclusive but not exhaustive can be

A: getting exactly three heads

B: getting one head and two tails

C: getting one tail and two heads

i.e.,

A = {HHH}

B = {HTT, THT, TTH}

C = {HHT, HTH, THH}

This is because A ∩ B = B ∩ C = C ∩ A = Φ, but A ∪ B ∪ C ≠ S

Question 6:

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events.

(i) A' (ii) not B (iii) A or B

(iv) A and B (v) A but not C (vi) B or C

(vii) B and C (viii)

$A \cap B' \cap C'$

Answer:

When two dice are thrown, the sample space is given by.

$$S = \{(x, y) : x, y = 1, 2, 3, 4, 5, 6\}$$

$$= \left\{\begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix}\right\}$$

Accordingly,

$$A = \left\{\begin{matrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3), \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$

$$B = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3), \\ (3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{matrix}\right\}$$

$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$$

$$\text{(i)} \quad A' = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3), \\ (3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{matrix}\right\} = B$$

$$\text{(ii)} \quad \text{Not } B = B' = \left\{\begin{matrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3), \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\} = A$$

(iii)

$$A \text{ or } B = A \cup B = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix} = S$$

(iv) $A \text{ and } B = A \cap B = \phi$

(v) $A \text{ but not } C = A - C$

$$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

(vi) $B \text{ or } C = B \cup C$

$$= \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2), \\ (2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

(vii) B and C = B ∩ C $\{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii)

$$C' = \begin{Bmatrix} (1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2), \\ (4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Question 7:

Refer to question 6 above, State true or false: (give reason for your **Answer**)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii) $A = B'$

(iv) A and C are mutually exclusive

(v) A and B are mutually exclusive

(vi) A', B', C are mutually exclusive and exhaustive.

Answer:

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3), \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$
$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3), \\ (3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$
$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$$

(i) It is observed that A ∩ B = Φ

∴ A and B are mutually exclusive.

Thus, the given statement is true.

(ii) It is observed that A ∩ B = Φ and A ∪ B = S

∴ A and B are mutually exclusive and exhaustive.

Thus, the given statement is true.

(iii) It is observed that

$$B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3), \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = A$$

Thus, the given statement is true.

(iv) It is observed that A ∩ C = {(2, 1), (2, 2), (2, 3), (4, 1)} ≠Φ

∴A and C are not mutually exclusive.

Thus, the given statement is false.

(v) $A \cap B' = A \cap A = A$

$\therefore\ A \cap B' \neq \phi$

∴A and B' are not mutually exclusive.

Thus, the given statement is false.

(vi) It is observed that $A' \cup B' \cup C = S$;

However, $B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \phi$

Therefore, events A', B' and C are not mutually exclusive and exhaustive.

Thus, the given statement is false.

Exercise 14.2

Question 1:

Which of the following cannot be valid assignment of probabilities for outcomes of sample space S = $\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	−0.1	0.2	0.3	0.4	−0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Answer:

(a)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.01	0.05	0.03	0.01	0.2	0.6

Here, each of the numbers *p*(ω*i*) is positive and less than 1.

Sum of probabilities

$$
\begin{aligned}
&= p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7) \\
&= 0.1+0.01+0.05+0.03+0.01+0.2+0.6 \\
&= 1
\end{aligned}
$$

Thus, the assignment is valid.

(b)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7

$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Here, each of the numbers $p(\omega i)$ is positive and less than 1.

Sum of probabilities

$$= p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7)$$
$$= \frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7} = 7\times\frac{1}{7} = 1$$

Thus, the assignment is valid.

(c)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.2	0.3	0.4	0.5	0.6	0.7

Here, each of the numbers $p(\omega i)$ is positive and less than 1.

Sum of probabilities

$$= p(\omega_1)+p(\omega_2)+p(\omega_3)+p(\omega_4)+p(\omega_5)+p(\omega_6)+p(\omega_7)$$
$$= 0.1+0.2+0.3+0.4+0.5+0.6+0.7$$
$$= 2.8 \neq 1$$

Thus, the assignment is not valid.

(d)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
−0.1	0.2	0.3	0.4	−0.2	0.1	0.3

Here, $p(\omega_1)$ and $p(\omega_5)$ are negative.

Hence, the assignment is not valid.

(e)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Here, $p(\omega_7) = \frac{15}{14} > 1$

Hence, the assignment is not valid.

Question 2:

A coin is tossed twice, what is the probability that at least one tail occurs?

Answer:

When a coin is tossed twice, the sample space is given by.

S = {HH, HT, TH, TT}

Let A be the event of the occurrence of at least one tail.

Accordingly, A = {HT, TH, TT}

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$
$$= \frac{n(A)}{n(S)}$$
$$= \frac{3}{4}$$

Question 3:

A die is thrown, find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Answer:

The sample space of the given experiment is given by

S = {1, 2, 3, 4, 5, 6}

(i) Let A be the event of the occurrence of a prime number.

Accordingly, A = {2, 3, 5}

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Let B be the event of the occurrence of a number greater than or equal to 3.
Accordingly, B = {3, 4, 5, 6}

$$\therefore P(B) = \frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}} = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(iii) Let C be the event of the occurrence of a number less than or equal to one.
Accordingly, C = {1}

$$\therefore P(C) = \frac{\text{Number of outcomes favourable to C}}{\text{Total number of possible outcomes}} = \frac{n(C)}{n(S)} = \frac{1}{6}$$

(iv) Let D be the event of the occurrence of a number greater than 6.

Accordingly, D = Φ

$$\therefore P(D) = \frac{\text{Number of outcomes favourable to D}}{\text{Total number of possible outcomes}} = \frac{n(D)}{n(S)} = \frac{0}{6} = 0$$

(v) Let E be the event of the occurrence of a number less than 6.

Accordingly, E = {1, 2, 3, 4, 5}

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{5}{6}$$

Question 4:

A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

Answer:

(a) When a card is selected from a pack of 52 cards, the number of possible outcomes is 52 i.e., the sample space contains 52 elements.

Therefore, there are 52 points in the sample space.

(b) Let A be the event in which the card drawn is an ace of spades.

Accordingly, $n(\text{A}) = 1$

$$\therefore \text{P}(\text{A}) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(\text{A})}{n(\text{S})} = \frac{1}{52}$$

(c) (i)Let E be the event in which the card drawn is an ace.

Since there are 4 aces in a pack of 52 cards, $n(\text{E}) = 4$

$$\therefore \text{P}(\text{E}) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \frac{n(\text{E})}{n(\text{S})} = \frac{4}{52} = \frac{1}{13}$$

(ii)Let F be the event in which the card drawn is black.

Since there are 26 black cards in a pack of 52 cards, $n(\text{F}) = 26$

$$\therefore \text{P}(\text{F}) = \frac{\text{Number of outcomes favourable to F}}{\text{Total number of possible outcomes}} = \frac{n(\text{F})}{n(\text{S})} = \frac{26}{52} = \frac{1}{2}$$

Question 5:

A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Answer:

Since the fair coin has 1 marked on one face and 6 on the other, and the die has six faces that are numbered 1, 2, 3, 4, 5, and 6, the sample space is given by

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Accordingly, $n(\text{S}) = 12$

(i) Let A be the event in which the sum of numbers that turn up is 3.

Accordingly, A = {(1, 2)}

$$\therefore \text{P}(\text{A}) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(\text{A})}{n(\text{S})} = \frac{1}{12}$$

(ii) Let B be the event in which the sum of numbers that turn up is 12.

Accordingly, B = {(6, 6)}

$$\therefore P(B)=\frac{\text{Number of outcomes favourable to B}}{\text{Total number of possible outcomes}}=\frac{n(B)}{n(S)}=\frac{1}{12}$$

Question 6:

There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Answer:

There are four men and six women on the city council.

As one council member is to be selected for a committee at random, the sample space contains 10 (4 + 6) elements.

Let A be the event in which the selected council member is a woman.

Accordingly, $n(A) = 6$

$$\therefore P(A)=\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}=\frac{n(A)}{n(S)}=\frac{6}{10}=\frac{3}{5}$$

Question 7:

A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Answer:

Since the coin is tossed four times, there can be a maximum of 4 heads or tails.

When 4 heads turns up, $\text{Re}1+\text{Re}1+\text{Re}1+\text{Re}1=\text{Rs }4$ is the gain.

When 3 heads and 1 tail turn up, Re 1 + Re 1 + Re 1 – Rs 1.50 = Rs 3 – Rs 1.50 = Rs 1.50 is the gain.

When 2 heads and 2 tails turns up, Re 1 + Re 1 – Rs 1.50 – Rs 1.50 = – Re 1, i.e., Re 1 is the loss.

When 1 head and 3 tails turn up, Re 1 – Rs 1.50 – Rs 1.50 – Rs 1.50 = – Rs 3.50, i.e., Rs 3.50 is the loss.

When 4 tails turn up, – Rs 1.50 – Rs 1.50 – Rs 1.50 – Rs 1.50 = – Rs 6.00, i.e., Rs 6.00 is the loss.

There are 2^4 = 16 elements in the sample space S, which is given by:

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT}

$\therefore n(S) = 16$

The person wins Rs 4.00 when 4 heads turn up, i.e., when the event {HHHH} occurs.

∴Probability (of winning Rs 4.00) = $\frac{1}{16}$

The person wins Rs 1.50 when 3 heads and one tail turn up, i.e., when the event {HHHT, HHTH, HTHH, THHH} occurs.

∴Probability (of winning Rs 1.50) = $\frac{4}{16} = \frac{1}{4}$

The person loses Re 1.00 when 2 heads and 2 tails turn up, i.e., when the event {HHTT, HTTH, TTHH, HTHT, THTH, THHT} occurs.

∴Probability (of losing Re 1.00) $= \frac{6}{16} = \frac{3}{8}$

The person loses Rs 3.50 when 1 head and 3 tails turn up, i.e., when the event {HTTT, THTT, TTHT, TTTH} occurs.

Probability (of losing Rs 3.50) = $\frac{4}{16} = \frac{1}{4}$

The person loses Rs 6.00 when 4 tails turn up, i.e., when the event {TTTT} occurs.

Probability (of losing Rs 6.00) = $\frac{1}{16}$

Question 8:

Three coins are tossed once. Find the probability of getting.

(i) 3 heads (ii) 2 heads (iii) at least 2 heads

(iv) at most 2 heads (v) no head (vi) 3 tails

(vii) exactly two tails (viii) no tail (ix) at most two tails.

Answer:

When three coins are tossed once, the sample space is given by

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴Accordingly, $n(S) = 8$

It is known that the probability of an event A is given by

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)}$$

(i) Let B be the event of the occurrence of 3 heads. Accordingly, B = {HHH}

∴P(B) = $\frac{n(B)}{n(S)} = \frac{1}{8}$

(ii) Let C be the event of the occurrence of 2 heads. Accordingly, C = {HHT, HTH, THH}

∴P(C) = $\frac{n(C)}{n(S)} = \frac{3}{8}$

(iii) Let D be the event of the occurrence of at least 2 heads.

Accordingly, D = {HHH, HHT, HTH, THH}

∴P(D) = $\frac{n(D)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iv) Let E be the event of the occurrence of at most 2 heads.

Accordingly, E = {HHT, HTH, THH, HTT, THT, TTH, TTT}

∴P(E) = $\frac{n(E)}{n(S)} = \frac{7}{8}$

(v) Let F be the event of the occurrence of no head.

Accordingly, F = {TTT}

∴P(F) = $\frac{n(F)}{n(S)} = \frac{1}{8}$

(vi) Let G be the event of the occurrence of 3 tails.

Accordingly, G = {TTT}

∴P(G) = $\frac{n(G)}{n(S)} = \frac{1}{8}$

(vii) Let H be the event of the occurrence of exactly 2 tails.

Accordingly, H = {HTT, THT, TTH}

∴P(H) = $\frac{n(H)}{n(S)} = \frac{3}{8}$

(viii) Let I be the event of the occurrence of no tail.

Accordingly, I = {HHH}

∴P(I) = $\frac{n(I)}{n(S)} = \frac{1}{8}$

(ix) Let J be the event of the occurrence of at most 2 tails.

Accordingly, I = {HHH, HHT, HTH, THH, HTT, THT, TTH}

∴P(J) = $\frac{n(J)}{n(S)} = \frac{7}{8}$

Question 9:

If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Answer:

It is given that P(A) = $\frac{2}{11}$.

Accordingly, P(not A) = 1 – P(A) $= 1 - \frac{2}{11} = \frac{9}{11}$

Question 10:

A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) an consonant.

Answer:

There are 13 letters in the word ASSASSINATION.

∴Hence, *n*(S) = 13

(i) There are 6 vowels in the given word.

∴Probability (vowel) = $\frac{6}{13}$

(ii) There are 7 consonants in the given word.

∴Probability (consonant) = $\frac{7}{13}$

Question 11:

In a lottery, person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [**Hint:** order of the numbers is not important.]

Answer:

Total number of ways in which one can choose six different numbers from 1 to 20 $= {}^{20}C_6 = \frac{\lfloor 20}{\lfloor 6 \lfloor 20-6} = \frac{\lfloor 20}{\lfloor 6 \lfloor 14} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 38760$

Hence, there are 38760 combinations of 6 numbers.

Out of these combinations, one combination is already fixed by the lottery committee.

∴ Required probability of winning the prize in the game = $\frac{1}{38760}$

Question 12:

Check whether the following probabilities P(A) and P(B) are consistently defined.

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Answer:

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

It is known that if E and F are two events such that E ⊂ F, then P(E) ≤ P(F).

However, here, P(A ∩ B) > P(A).

Hence, P(A) and P(B) are not consistently defined.

(ii)P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

It is known that if E and F are two events such that E ⊂ F, then P(E) ≤ P(F).

Here, it is seen that P(A ∪ B) > P(A) and P(A ∪ B) > P(B).

Hence, P(A) and P(B) are consistently defined.

Question 13:

Fill in the blanks in following table:

	P(A)	P(B)	P(A ∩ B)	P(A ∪ B)
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	...
(ii)	0.35	...	0.25	0.6
(iii)	0.5	0.35	...	0.7

Answer:

(i) Here, $P(A)=\frac{1}{3}, P(B)=\frac{1}{5}, P(A\cap B)=\frac{1}{15}$

We know that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\therefore P(A\cup B)=\frac{1}{3}+\frac{1}{5}-\frac{1}{15}=\frac{5+3-1}{15}=\frac{7}{15}$$

(ii) Here, P(A) = 0.35, P(A ∩ B) = 0.25, P(A ∪ B) = 0.6

We know that P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

∴0.6 = 0.35 + P(B) − 0.25

⇒ P(B) = 0.6 − 0.35 + 0.25

⇒ P(B) = 0.5

(iii)Here, P(A) = 0.5, P(B) = 0.35, P(A ∪ B) = 0.7

We know that P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

∴0.7 = 0.5 + 0.35 − P(A ∩ B)

⇒ P(A ∩ B) = 0.5 + 0.35 − 0.7

⇒ P(A ∩ B) = 0.15

Question 14:

Given P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

Answer:

Here, P(A) = $\frac{3}{5}$, P(B) = $\frac{1}{5}$

For mutually exclusive events A and B,

P(A or B) = P(A) + P(B)

∴ P(A or B) $=\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

Question 15:

If E and F are events such that P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$, find:(i) P(E or F), (ii) P(not E and not F).

Answer:

Here, P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$, and P(E and F) = $\frac{1}{8}$

(i) We know that P(E or F) = P(E) + P(F) − P(E and F)

∴P(E or F) = $\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{2+4-1}{8}=\frac{5}{8}$

(ii) From (i), P(E or F) = P (E ∪ F) = $\frac{5}{8}$

We have $(E\cup F)'=(E'\cap F')$ [By De Morgan's law]

$\therefore P(E'\cap F')=P(E\cup F)'$

Now, $P(E\cup F)'=1-P(E\cup F)=1-\frac{5}{8}=\frac{3}{8}$

$\therefore P(E'\cap F')=\frac{3}{8}$

Thus, P(not E and not F) $=\frac{3}{8}$

Question 16:

Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.

Answer:

It is given that P (not E or not F) = 0.25

$$\text{i.e., } P(E' \cup F') = 0.25$$

$$\Rightarrow P(E \cap F)' = 0.25 \qquad [E' \cup F' = (E \cap F)']$$

$$\text{Now, } P(E \cap F) = 1 - P(E \cap F)'$$

$$\Rightarrow P(E \cap F) = 1 - 0.25$$

$$\Rightarrow P(E \cap F) = 0.75 \neq 0$$

$$\Rightarrow E \cap F \neq \phi$$

Thus, E and F are not mutually exclusive.

Question 17:

A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P(not A), (ii) P (not B) and (iii) P(A or B).

Answer:

It is given that P(A) = 0.42, P(B) = 0.48, P(A and B) = 0.16

(i) P(not A) = 1 – P(A) = 1 – 0.42 = 0.58

(ii) P(not B) = 1 – P(B) = 1 – 0.48 = 0.52

(iii) We know that P(A or B) = P(A) + P(B) – P(A and B)

∴ P(A or B) = 0.42 + 0.48 – 0.16 = 0.74

Question 18:

In Class XI of a school 40% of the student's study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Answer:

Let A be the event in which the selected student studies Mathematics and B be the event in which the selected student studies Biology.

Accordingly, P(A) = 40% = $\frac{40}{100} = \frac{2}{5}$

P(B) = 30% $= \frac{30}{100} = \frac{3}{10}$

P(A and B) = 10% $= \frac{10}{100} = \frac{1}{10}$

We know that P(A or B) = P(A) + P(B) â€“ P(A and B)

$$\therefore P(A \text{ or } B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = 0.6$$

Thus, the probability that the selected student will be studying Mathematics or Biology is 0.6.

Question 19:

In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Answer:

Let A and B be the events of passing first and second examinations respectively.

Accordingly, P(A) = 0.8, P(B) = 0.7 and P(A or B) = 0.95

We know that P(A or B) = P(A) + P(B) – P(A and B)

∴0.95 = 0.8 + 0.7 – P(A and B)

⇒ P(A and B) = 0.8 + 0.7 – 0.95 = 0.55

Thus, the probability of passing both the examinations is 0.55.

Question 20:

The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Answer:

Let A and B be the events of passing English and Hindi examinations respectively.

Accordingly, P(A and B) = 0.5, P(not A and not B) = 0.1, i.e., $P(A' \cap B') = 0.1$

P(A) = 0.75

$$\text{Now, } (A \cup B)' = (A' \cap B') \qquad [\text{De Morgan's law}]$$
$$\therefore P(A \cup B)' = P(A' \cap B') = 0.1$$
$$P(A \cup B) = 1 - P(A \cup B)' = 1 - 0.1 = 0.9$$

We know that P(A or B) = P(A) + P(B) – P(A and B)

∴0.9 = 0.75 + P(B) – 0.5

⇒ P(B) = 0.9 – 0.75 + 0.5

⇒ P(B) = 0.65

Thus, the probability of passing the Hindi examination is 0.65.

Question 21:

In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) The student has opted neither NCC nor NSS.

(iii) The student has opted NSS but not NCC.

Answer:

Let A be the event in which the selected student has opted for NCC and B be the event in which the selected student has opted for NSS.

Total number of students = 60

Number of students who have opted for NCC = 30

∴ P(A) = $\frac{30}{60} = \frac{1}{2}$

Number of students who have opted for NSS = 32

$$\therefore P(B) = \frac{32}{60} = \frac{8}{15}$$

Number of students who have opted for both NCC and NSS = 24

$$\therefore P(\text{A and B}) = \frac{24}{60} = \frac{2}{5}$$

(i) We know that P(A or B) = P(A) + P(B) – P(A and B)

$$\therefore P(\text{A or B}) = \frac{1}{2} + \frac{8}{15} - \frac{2}{5} = \frac{15+16-12}{30} = \frac{19}{30}$$

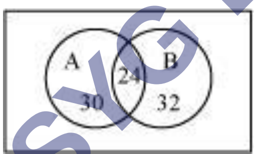
Thus, the probability that the selected student has opted for NCC or NSS is $\frac{19}{30}$.

(ii)

$$\begin{aligned} & P(\text{not A and not B}) \\ & = P(A' \text{ and } B') \\ & = P(A' \cap B') \\ & = P(A \cup B)' \qquad \left[(A' \cap B') = (A \cup B)' \text{ (by De Morgan's law)}\right] \\ & = 1 - P(A \cup B) \\ & = 1 - P(\text{A or B}) \\ & = 1 - \frac{19}{30} \\ & = \frac{11}{30} \end{aligned}$$

Thus, the probability that the selected students has neither opted for NCC nor NSS is $\frac{11}{30}$.

(iii) The given information can be represented by a Venn diagram as



It is clear that

Number of students who have opted for NSS but not NCC

= $n(B - A) = n(B) - n(A \cap B) = 32 - 24 = 8$

Thus, the probability that the selected student has opted for NSS but not for NCC

= $\frac{8}{60}=\frac{2}{15}$

Miscellaneous Exercise

Question 1:

A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) all will be blue? (ii) atleast one will be green?

Answer:

Total number of marbles = 10 + 20 + 30 = 60

Number of ways of drawing 5 marbles from 60 marbles = ${}^{60}C_5$

(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.

5 blue marbles can be drawn from 20 blue marbles in ${}^{20}C_5$ ways.

∴Probability that all marbles will be blue = $\frac{{}^{20}C_5}{{}^{60}C_5}$

(ii) Number of ways in which the drawn marble is not green = ${}^{(20+10)}C_5 = {}^{30}C_5$

∴Probability that no marble is green = $\frac{{}^{30}C_5}{{}^{60}C_5}$

∴Probability that at least one marble is green = $1-\frac{{}^{30}C_5}{{}^{60}C_5}$

Question 2:

4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Answer:

Number of ways of drawing 4 cards from 52 cards = ${}^{52}C_4$

In a deck of 52 cards, there are 13 diamonds and 13 spades.

â^´Number of ways of drawing 3 diamonds and one spade = $^{13}C_3 \times {}^{13}C_1$

Thus, the probability of obtaining 3 diamonds and one spade = $\frac{^{13}C_3 \times {}^{13}C_1}{^{52}C_4}$

Question 3:

A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

(i) P(2) (ii) P(1 or 3) (iii) P(not 3)

Answer:

Total number of faces = 6

(i) Number faces with number '2' = 3

$\therefore P(2) = \frac{3}{6} = \frac{1}{2}$

(ii) P (1 or 3) = P (not 2) = 1 − P (2) $= 1 - \frac{1}{2} = \frac{1}{2}$

(iii) Number of faces with number '3' = 1

$\therefore P(3) = \frac{1}{6}$

Thus, $P(\text{not } 3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$

Question 4:

In a certain lottery, 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets?

Answer:

Total number of tickets sold = 10,000

Number prizes awarded = 10

(i) If we buy one ticket, then

P (getting a prize) = $\frac{10}{10000} = \frac{1}{1000}$

∴P (not getting a prize) = $1-\frac{1}{1000}=\frac{999}{1000}$

(ii) If we buy two tickets, then

Number of tickets not awarded = 10,000 – 10 = 9990

P (not getting a prize) = $\frac{^{9990}C_2}{^{10000}C_2}$

(iii) If we buy 10 tickets, then

P (not getting a prize) = $\frac{^{9990}C_{10}}{^{10000}C_{10}}$

Question 5:

Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) you both enter the same sections?

(b) you both enter the different sections?

Answer:

My friend and I are among the 100 students.

Total number of ways of selecting 2 students out of 100 students = $^{100}C_2$

(a) The two of us will enter the same section if both of us are among 40 students or among 60 students.

∴ Number of ways in which both of us enter the same section = $^{40}C_2+{}^{60}C_2$

∴ Probability that both of us enter the same section

$$=\frac{^{40}C_2+{}^{60}C_2}{^{100}C_2}=\frac{\frac{\lfloor 40}{\lfloor 2\lfloor 38}+\frac{\lfloor 60}{\lfloor 2\lfloor 58}}{\frac{\lfloor 100}{\lfloor 2\lfloor 98}}=\frac{(39\times 40)+(59\times 60)}{99\times 100}=\frac{17}{33}$$

(b) P(we enter different sections)

= 1 – P(we enter the same section)

$= 1-\frac{17}{33}=\frac{16}{33}$

Question 6:

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Answer:

Let L_1, L_2, L_3 be three letters and E_1, E_2, and E_3 be their corresponding envelops respectively.

There are 6 ways of inserting 3 letters in 3 envelops. These are as follows:

$$\left.\begin{array}{l} L_1E_1, L_2E_3, L_3E_2 \\ L_2E_2, L_1E_3, L_3E_1 \\ L_3E_3, L_1E_2, L_2E_1 \\ L_1E_1, L_2E_2, L_3E_3 \end{array}\right]$$
$$\begin{array}{l} L_1E_2, L_2E_3, L_3E_1 \\ L_1E_3, L_2E_1, L_3E_2 \end{array}$$

There are 4 ways in which at least one letter is inserted in a proper envelope.

Thus, the required probability is $\frac{4}{6}=\frac{2}{3}$.

Question 7:

A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) = 0.35.

Find (i) P(A ∩ B) (ii) P(A' ∩ B') (iii) P(A ∩ B') (iv) P(B ∩ A')

Answer:

It is given that P(A) = 0.54, P(B) = 0.69, P(A ∩ B) = 0.35

(i) We know that P (A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴P (A ∪ B) = 0.54 + 0.69 – 0.35 = 0.88

(ii) A' ∩ B' = (A ∪ B)' [by De Morgan's law]

∴P(A' ∩ B') = P(A ∪ B)' = 1 – P(A ∪ B) = 1 – 0.88 = 0.12

(iii) P(A ∩ B') = P(A) – P(A ∩ B)

= 0.54 – 0.35

= 0.19

(iv) We know that $n(B \cap A') = n(B) - n(A \cap B)$

$$\Rightarrow \frac{n(B \cap A')}{n(S)} = \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$
$$\therefore P(B \cap A') = P(B) - P(A \cap B)$$
$$\therefore P(B \cap A') = 0.69 - 0.35 = 0.34$$

Question 8:

From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Answer:

Let E be the event in which the spokesperson will be a male and F be the event in which the spokesperson will be over 35 years of age.

Accordingly, P(E) = $\frac{3}{5}$ and P(F) = $\frac{2}{5}$

Since there is only one male who is over 35 years of age,

$$P(E \cap F) = \frac{1}{5}$$

We know that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$\therefore P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$$

Thus, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

Question 9:

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) the repetition of digits is not allowed?

Answer:

(i) When the digits are repeated

Since four-digit numbers greater than 5000 are formed, the leftmost digit is either 7 or 5.

The remaining 3 places can be filled by any of the digits 0, 1, 3, 5, or 7 as repetition of digits is allowed.

∴Total number of 4-digit numbers greater than 5000 = 2 × 5 × 5 × 5 – 1

= 250 – 1 = 249

[In this case, 5000 can not be counted; so 1 is subtracted]

A number is divisible by 5 if the digit at its units place is either 0 or 5.

∴Total number of 4-digit numbers greater than 5000 that are divisible by 5 = 2 × 5 × 5 × 2 – 1 = 100 – 1 = 99

Thus, the probability of forming a number divisible by 5 when the digits are repeated is $= \frac{99}{249} = \frac{33}{83}$.

(ii) When repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or 7.

The remaining 3 places can be filled with any of the remaining 4 digits.

∴Total number of 4-digit numbers greater than 5000 = 2 × 4 × 3 × 2

= 48

When the digit at the thousands place is 5, the units place can be filled only with 0 and the tens and hundreds places can be filled with any two of the remaining 3 digits.

∴Here, number of 4-digit numbers starting with 5 and divisible by 5

$= 3 \times 2 = 6$

When the digit at the thousands place is 7, the units place can be filled in two ways (0 or 5) and the tens and hundreds places can be filled with any two of the remaining 3 digits.

∴ Here, number of 4-digit numbers starting with 7 and divisible by 5

$= 1 \times 2 \times 3 \times 2 = 12$

∴Total number of 4-digit numbers greater than 5000 that are divisible by 5 = 6 + 12 = 18

Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed is $\frac{18}{48} = \frac{3}{8}$.

Question 10:

The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Answer:

The number lock has 4 wheels, each labelled with ten digits i.e., from 0 to 9.

Number of ways of selecting 4 different digits out of the 10 digits = ${}^{10}C_4$

Now, each combination of 4 different digits can be arranged in $\lfloor 4$ ways.

∴Number of four digits with no repetitions = ${}^{10}C_4 \times \lfloor 4 = \frac{\lfloor 10}{\lfloor 4 \lfloor 6} \times \lfloor 4 = 7 \times 8 \times 9 \times 10 = 5040$

There is only one number that can open the suitcase.

Thus, the required probability is $\frac{1}{5040}$.