Chapter 3

Trigonometric Functions

Exercise 3.1

Question 1:

Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) – 47° 30' (iii) 240° (iv) 520°

Answer:

(i) 25°

We know that 180° = π radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) −47° 30'

−47° 30' = $-47\frac{1}{2}$ degree [1° = 60']

$= \frac{-95}{2}$ degree

Since 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right) \pi \text{ radian} = \frac{-19}{72} \pi \text{ radian}$$

$$\therefore -47° 30' = \frac{-19}{72} \pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3} \pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520^\circ = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Question 2:

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ (ii) – 4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Answer:

(i) $\frac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

$$= 39^\circ + \frac{3 \times 60}{8} \text{ min utes} \qquad [1^\circ = 60']$$

$$= 39^\circ + 22' + \frac{1}{2} \text{ min utes}$$

$$= 39^\circ 22' 30'' \qquad [1' = 60'']$$

(ii) – 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ degree} = \frac{180 \times 7(-4)}{22} \text{ degree}$$

$$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$$

$$= -229° + \frac{1 \times 60}{11} \text{ minutes} \qquad [1° = 60']$$

$$= -229° + 5' + \frac{5}{11} \text{ minutes}$$

$$= -229°5'27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Question 3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer:

Number of revolutions made by the wheel in 1 minute = 360

∴Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,

12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Answer:

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, forr = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$
$$= \frac{126}{10} \text{ degree} = 12\frac{3}{5} \text{ degree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is 12°36'.

Question 5:

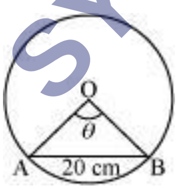
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Answer:

Diameter of the circle = 40 cm

∴Radius (*r*) of the circle = $\frac{40}{2}$ cm = 20 cm

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

$\therefore \theta = 60° = \frac{\pi}{3}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm.

Question 6:

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer:

Let the radii of the two circles be r_1 and r_2. Let an arc of length *l* subtend an angle of 60° at the centre of the circle of radius r_1, while let an arc of length *l* subtend an angle of 75° at the centre of the circle of radius r_2.

Now, 60° = $\frac{\pi}{3}$ radian and 75° = $\frac{5\pi}{12}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$.

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

Question 7:

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Answer:

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then $\theta = \frac{l}{r}$.

It is given that *r* = 75 cm

(i) Here, *l* = 10 cm

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) Here, *l* = 15 cm

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) Here, *l* = 21 cm

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$

Exercise 3.2

Question 1:

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, *x* lies in third quadrant.

Answer:

$$\cos x = -\frac{1}{2}$$

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$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since x lies in the 3rd quadrant, the value of sin x will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Question 2:

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, x lies in second quadrant.

Answer:

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

Since x lies in the 2nd quadrant, the value of cos x will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

Question 3:

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, x lies in third quadrant.

Answer:

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of sec x will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 4:

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Answer:

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm\frac{12}{13}$$

Since x lies in the 4th quadrant, the value of sin x will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Question 5:

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, x lies in second quadrant.

Answer:

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since x lies in the 2nd quadrant, the value of sec x will be negative.

∴sec x = $-\frac{13}{12}$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Question 6:

Find the value of the trigonometric function sin 765°

Answer:

It is known that the values of sin x repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin\left(2 \times 360° + 45°\right) = \sin 45° = \frac{1}{\sqrt{2}}$$

Question 7:

Find the value of the trigonometric function cosec (−1410°)

Answer:

It is known that the values of cosec *x* repeat after an interval of 2π or 360°.

$$\begin{aligned}\therefore \text{cosec}\ (-1410°) &= \text{cosec}(-1410° + 4 \times 360°) \\ &= \text{cosec}\ (-1410° + 1440°) \\ &= \text{cosec}30° = 2\end{aligned}$$

Question 8:

Find the value of the trigonometric function $\tan \frac{19\pi}{3}$

Answer:

It is known that the values of tan *x* repeat after an interval of π or 180°.

$$\therefore \tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \tan 60° = \sqrt{3}$$

Question 9:

Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

Answer:

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Question 10:

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

Answer:

It is known that the values of cot *x* repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right) = \cot \frac{\pi}{4} = 1$$

Exercise 3.3

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Answer:

L.H.S. = $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$
$$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$$
$$=\text{R.H.S.}$$

Question 2:

Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Answer:

L.H.S. = $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$
$$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$
$$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$$
$$=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}$$
$$=\text{R.H.S.}$$

Question 3:

Prove that $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Answer:

L.H.S. = $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$$=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$$
$$=3+\operatorname{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$$
$$=3+2+1=6$$
$$=\text{R.H.S}$$

Question 4:

Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Answer:

L.H.S = $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2$$
$$=2\left\{\sin\frac{\pi}{4}\right\}^2+2\times\frac{1}{2}+8$$
$$=2\left(\frac{1}{\sqrt{2}}\right)^2+1+8$$
$$=1+1+8$$
$$=10$$
$$=\text{R.H.S}$$

Question 5:

Find the value of:

(i) sin 75°

(ii) tan 15°

Answer:

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[sin ($x + y$) = sin x cos y + cos x sin y]

$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° – 30°)

$$=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ} \qquad \left[\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$$

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{4-2\sqrt{3}}{3-1}=2-\sqrt{3}$$

Question 6:

Prove that: $$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

Answer:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$\left[\begin{array}{l}\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)\\ -2\sin A\sin B=\cos(A+B)-\cos(A-B)\end{array}\right]$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x + y)\right]$$

$$= \sin(x + y)$$

$$= \text{R.H.S}$$

Question 7:

Prove that: $\dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\dfrac{1 + \tan x}{1 - \tan x}\right)^2$

Answer:

It is known that $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

∴L.H.S. = $\dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \dfrac{\left(\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right)} = \dfrac{\left(\dfrac{1 + \tan x}{1 - \tan x}\right)}{\left(\dfrac{1 - \tan x}{1 + \tan x}\right)} = \left(\dfrac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S.}$

Question 8:

Prove that $\dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$

Answer:

$$\text{L.H.S.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$$= \cot^2 x$$

$$= \text{R.H.S.}$$

Question 9:

$$\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$$

Answer:

L.H.S. = $\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$

$$= \sin x \cos x [\tan x + \cot x]$$

$$= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= (\sin x \cos x)\left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right]$$

$$= 1 = \text{R.H.S.}$$

Question 10:

Prove that sin $(n + 1)x$ sin $(n + 2)x$ + cos $(n + 1)x$ cos $(n + 2)x$ = cos x

Answer:

L.H.S. = sin $(n + 1)x$ sin$(n + 2)x$ + cos $(n + 1)x$ cos$(n + 2)x$

$$=\frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x\right]$$

$$=\frac{1}{2}\left[\begin{array}{l}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{array}\right]$$

$$\left[\begin{array}{l}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{array}\right]$$

$$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$

$$=\cos(-x)=\cos x=\text{R.H.S.}$$

Question 11:

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Answer:

It is known that $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$.

$\therefore$L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

$$=\text{R.H.S.}$$

Question 12:

Prove that $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

Answer:

It is known that $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

∴L.H.S. = $\sin^2 6x - \sin^2 4x$

= $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

= $(2\sin 5x\cos x)(2\cos 5x\sin x)$

= $(2\sin 5x\cos 5x)(2\sin x\cos x)$

= $\sin 10x\sin 2x$

= R.H.S.

Question 13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x\sin 8x$

Answer:

It is known that $\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

∴L.H.S. = $\cos^2 2x - \cos^2 6x$

= $(\cos 2x + \cos 6x)(\cos 2x - 6x)$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

= $[2\cos 4x\cos 2x][-2\sin 4x(-\sin 2x)]$

= $(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

= $\sin 8x\sin 4x$

= R.H.S.

Question 14:

Prove that sin 2*x* + 2sin 4*x* + sin 6*x* = 4cos^2 *x* sin 4*x*

Answer:

L.H.S. = sin 2*x* + 2 sin 4*x* + sin 6*x*

= [sin 2*x* + sin 6*x*] + 2 sin 4*x*

$$=\left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

= 2 sin 4*x* cos (â€" 2*x*) + 2 sin 4*x*

= 2 sin 4*x* cos 2*x* + 2 sin 4*x*

= 2 sin 4*x* (cos 2*x* + 1)

= 2 sin 4*x* (2 cos^2 *x* â€" 1 + 1)

= 2 sin 4*x* (2 cos^2 *x*)

= 4cos^2 *x* sin 4*x*

= R.H.S.

Question 15:

Prove that cot 4*x* (sin 5*x* + sin 3*x*) = cot *x* (sin 5*x* – sin 3*x*)

Answer:

L.H.S = cot 4*x* (sin 5*x* + sin 3*x*)

$$=\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

= 2 cos 4*x* cos *x*

R.H.S. = cot *x* (sin 5*x* â€" sin 3*x*)

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$
$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$
$$= \frac{\cos x}{\sin x}\left[2\cos 4x \sin x\right]$$

= 2 cos 4*x*. cos *x*

L.H.S. = R.H.S.

Question 16:

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Answer:

It is known that

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S = $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$
$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$
$$= -\frac{\sin 2x}{\cos 10x}$$
$$= \text{R.H.S.}$$

Question 17:

Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Answer:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\quad \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. = $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$=\frac{\sin 4x}{\cos 4x}$$

$$=\tan 4x=\text{R.H.S.}$$

Question 18:

Prove that $\dfrac{\sin x-\sin y}{\cos x+\cos y}=\tan\dfrac{x-y}{2}$

Answer:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\quad \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. = $\dfrac{\sin x-\sin y}{\cos x+\cos y}$

$$=\frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$=\tan\left(\frac{x-y}{2}\right)=\text{R.H.S.}$$

Question 19:

Prove that $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Answer:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\quad \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$
$$=\frac{\sin 2x}{\cos 2x}$$
$$=\tan 2x$$
$$=\text{R.H.S}$$

Question 20:

Prove that $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Answer:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\quad \cos^2 A-\sin^2 A=\cos 2A$$

∴L.H.S. = $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

$$=\frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$
$$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$$
$$=-2\times(-\sin x)$$
$$=2\sin x=\text{R.H.S.}$$

Question 21:

Prove that $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Answer:

L.H.S. = $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$

$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}$$

$$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$$=\cot 3x=\text{R.H.S.}$$

Question 22:

Prove that cot *x* cot 2*x* – cot 2*x* cot 3*x* – cot 3*x* cot *x* = 1

Answer:

L.H.S. = cot *x* cot 2*x* – cot 2*x* cot 3*x* – cot 3*x* cot *x*

= cot *x* cot 2*x* – cot 3*x* (cot 2*x* + cot *x*)

= cot *x* cot 2*x* – cot (2*x* + *x*) (cot 2*x* + cot *x*)

$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$

$$\left[\because \cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]$$

= cot *x* cot 2*x* – (cot 2*x* cot *x* – 1)

= 1 = R.H.S.

Question 23:

Prove that $\tan 4x = \dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

Answer:

It is known that $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

∴L.H.S. = tan 4x = tan 2(2x)

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

Question 24:

Prove that cos 4x = 1 – 8$\sin^2 x \cos^2 x$

Answer:

L.H.S. = cos 4x

$= \cos 2(2x)$

$= 1 - 2\sin^2 2x$ [$\cos 2A = 1 - 2\sin^2 A$]

$= 1 - 2(2\sin x \cos x)^2$ [$\sin 2A = 2\sin A \cos A$]

$= 1 - 8\sin^2 x \cos^2 x$

= R.H.S.

Question 25:

Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Answer:

L.H.S. = $\cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x$ [$\cos 3A = 4\cos^3 A - 3\cos A$]

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$ [$\cos 2x = 2\cos^2 x - 1$]

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

= R.H.S.