Chapter 4
Complex Numbers
Exercise 4.1

Question 1:

Express the given complex number in the form *a* + *ib*: $(5i)\left(-\frac{3}{5}i\right)$

Answer:

$$\begin{aligned}(5i)\left(\frac{-3}{5}i\right) &= -5\times\frac{3}{5}\times i\times i\\ &= -3i^2\\ &= -3(-1) \qquad \left[i^2=-1\right]\\ &= 3\end{aligned}$$

Question 2:

Express the given complex number in the form *a* + *ib*: $i^9 + i^{19}$

Answer:

$$\begin{aligned}i^9+i^{19} &= i^{4\times2+1}+i^{4\times4+3}\\ &= \left(i^4\right)^2\cdot i+\left(i^4\right)^4\cdot i^3\\ &= 1\times i+1\times(-i) \qquad \left[i^4=1,\ i^3=-i\right]\\ &= i+(-i)\\ &= 0\end{aligned}$$

Question 3:

Express the given complex number in the form *a* + *ib*: i^{-39}

Answer:

$$\begin{aligned} i^{-39} &= i^{-4\times 9-3} = \left(i^4\right)^{-9} \cdot i^{-3} \\ &= (1)^{-9} \cdot i^{-3} \qquad \left[i^4 = 1\right] \\ &= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right] \\ &= \frac{-1}{i} \times \frac{i}{i} \\ &= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right] \end{aligned}$$

Question 4:

Express the given complex number in the form *a* + *ib*: 3(7 + *i*7) + *i*(7 + *i*7)

Answer:

$$\begin{aligned} 3(7+i7)+i(7+i7) &= 21+21i+7i+7i^2 \\ &= 21+28i+7\times(-1) \qquad \left[\because i^2 = -1\right] \\ &= 14+28i \end{aligned}$$

Question 5:

Express the given complex number in the form *a* + *ib*: (1 − *i*) − (−1 + *i*6)

Answer:

$$\begin{aligned} (1-i)-(-1+i6) &= 1-i+1-6i \\ &= 2-7i \end{aligned}$$

Question 6:

Express the given complex number in the form *a* + *ib*: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Question 7:

Express the given complex number in the form *a* + *ib*: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Answer:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Question 8:

Express the given complex number in the form *a* + *ib*: $(1-i)^4$

Answer:

$$(1-i)^4=\left[(1-i)^2\right]^2$$
$$=\left[1^2+i^2-2i\right]^2$$
$$=\left[1-1-2i\right]^2$$
$$=(-2i)^2$$
$$=(-2i)\times(-2i)$$
$$=4i^2=-4 \qquad \left[i^2=-1\right]$$

Question 9:

Express the given complex number in the form *a* + *ib*: $\left(\frac{1}{3}+3i\right)^3$

Answer:

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad [i^3=-i]$$
$$=\frac{1}{27}-27i+i-9 \qquad [i^2=-1]$$
$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$
$$=\frac{-242}{27}-26i$$

Question 10:

Express the given complex number in the form *a* + *ib*: $\left(-2-\frac{1}{3}i\right)^3$

Answer:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad [i^3=-i]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad [i^2=-1]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number 4 – 3*i*

Answer:

Let z = 4 – 3*i*

Then,

$\bar{z}$ = 4 + 3*i* and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of 4 – 3*i* is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Answer:

Let z = $\sqrt{5}+3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number –*i*

Answer:

Let z = –*i*

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of –*i* is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

Question 14:

Express the following expression in the form of *a* + *ib*.

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Answer:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Miscellaneous Execrise

Question 1:

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Answer:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$

$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$

$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$

$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$

$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$

$$=\left[-1+\frac{i}{i^2}\right]^3$$

$$=\left[-1-i\right]^3$$

$$=(-1)^3\left[1+i\right]^3$$

$$=-\left[1^3+i^3+3\cdot1\cdot i(1+i)\right]$$

$$=-\left[1+i^3+3i+3i^2\right]$$

$$=-\left[1-i+3i-3\right]$$

$$=-\left[-2+2i\right]$$

$$=2-2i$$

Question 2:

For any two complex numbers z_1 and z_2, prove that

Re (z_1z_2) = Re z_1 Re z_2 − Im z_1 Im z_2

Answer:

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

Question 3:

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Answer:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$

$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad \text{[On multiplying numerator and denominator by } (14+5i)]$$
$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$
$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

This is the required standard form.

Question 4:

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Answer:

$$x - iy = \sqrt{\frac{a - ib}{c - id}}$$

$$= \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}} \text{ [On multiplying numerator and denominator by } (c + id)]$$

$$= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}}$$

$$\therefore (x - iy)^2 = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

$$\Rightarrow x^2 - y^2 - 2ixy = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2}, \quad -2xy = \frac{ad - bc}{c^2 + d^2} \qquad (1)$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \qquad [\text{Using } (1)]$$

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}$$

$$= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}$$

$$= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}$$

$$= \frac{a^2 + b^2}{c^2 + d^2}$$

Hence, proved.

Question 5:

If $z_1 = 2 - i,\ z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$.

Answer:

$$z_1 = 2 - i,\ z_2 = 1 + i$$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Thus, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

Question 6:

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x+1)^2}$

Answer:

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$

$$= \frac{x^2 + i^2 + 2xi}{2x^2+1}$$

$$= \frac{x^2 - 1 + i2x}{2x^2+1}$$

$$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$

On comparing real and imaginary parts, we obtain

$$a = \frac{x^2 - 1}{2x^2 + 1} \text{ and } b = \frac{2x}{2x^2 + 1}$$

$$\therefore a^2 + b^2 = \left(\frac{x^2 - 1}{2x^2 + 1}\right)^2 + \left(\frac{2x}{2x^2 + 1}\right)^2$$

$$= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x+1)^2}$$

$$= \frac{x^4 + 1 + 2x^2}{(2x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

$$\therefore a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

Hence, proved.

Question 7:

Let $z_1 = 2 - i,\ z_2 = -2 + i$. Find

(i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) $\text{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$

Answer:

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\bar{z}_1 = 2 + i$

$\therefore \frac{z_1 z_2}{\bar{z}_1} = \frac{-3 + 4i}{2 + i}$

On multiplying numerator and denominator by (2 – *i*), we obtain

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{2^2 + 1^2}$$

$$= \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

On comparing real parts, we obtain

$$\text{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$$

(ii) $\frac{1}{z_1\overline{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2+(1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$$\text{Im}\left(\frac{1}{z_1\overline{z}_1}\right) = 0$$

Question 8:

Find the real numbers *x* and *y* if (*x* – *iy*) (3 + 5*i*) is the conjugate of –6 – 24*i*.

Answer:

Let $z = (x - iy)(3 + 5i)$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$
$$\therefore \overline{z} = (3x + 5y) - i(5x - 3y)$$

It is given that, $\overline{z} = -6 - 24i$

$$\therefore (3x + 5y) - i(5x - 3y) = -6 - 24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6 \quad \text{... (i)}$$
$$5x - 3y = 24 \quad \text{... (ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$9x + 15y = -18$$
$$25x - 15y = 120$$
$$34x = 102$$
$$\therefore x = \frac{102}{34} = 3$$

Putting the value of *x* in equation (i), we obtain

$$3(3)+5y=-6$$
$$\Rightarrow 5y=-6-9=-15$$
$$\Rightarrow y=-3$$

Thus, the values of *x* and *y* are 3 and –3 respectively.

Question 9:

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Answer:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$
$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$
$$=\frac{4i}{2}=2i$$
$$\therefore \left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

Question 10:

If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$.

Answer:

$$(x+iy)^3=u+iv$$
$$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$$
$$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$$
$$\Rightarrow x^3-iy^3+3x^2yi-3xy^2=u+iv$$
$$\Rightarrow \left(x^3-3xy^2\right)+i\left(3x^2y-y^3\right)=u+iv$$

On equating real and imaginary parts, we obtain

$$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$$

$$\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$

$$= \frac{x\left(x^2 - 3y^2\right)}{x} + \frac{y\left(3x^2 - y^2\right)}{y}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4\left(x^2 - y^2\right)$$

$$\therefore \frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$$

Hence, proved.

Question 11:

If α and β are different complex numbers with $|\beta|$ = 1, then find $\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$.

Answer:

Let α = *a* + *ib* and β = *x* + *iy*

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2 + y^2} = 1$$

$$\Rightarrow x^2 + y^2 = 1 \qquad \text{... (i)}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$=\frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$$

$$=\frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$=\frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$=\frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$=\frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad [\text{Using } (1)]$$

$$=1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

Question 12:

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

Answer:

$$|1-i|^x = 2^x$$
$$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$
$$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$$
$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$
$$\Rightarrow \frac{x}{2} = x$$
$$\Rightarrow x = 2x$$
$$\Rightarrow 2x - x = 0$$
$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Question 13:

If (a + ib) (c + id) (e + if) (g + ih) = A + iB, then show that

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Answer:

$$(a+ib)(c+id)(e+if)(g+ih) = A + iB$$

$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$
$$\Rightarrow |(a+ib)|\times|(c+id)|\times|(e+if)|\times|(g+ih)| = |A+iB| \qquad [|z_1z_2| = |z_1||z_2|]$$
$$\Rightarrow \sqrt{a^2+b^2}\times\sqrt{c^2+d^2}\times\sqrt{e^2+f^2}\times\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

On squaring both sides, we obtain

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Hence, proved.

Question 14:

If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

Answer:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$\therefore m = 4k$, where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of *m* is 4 (= 4 × 1).