Chapter 5
Linear Inequalities
Exercise 5.1

Question 1:

Solve 24x < 100, when (i) x is a natural number (ii) x is an integer

Answer:

The given inequality is 24x < 100.

$$24x < 100$$
$$\Rightarrow \frac{24x}{24} < \frac{100}{24} \quad [\text{Dividing both sides by same positive number}]$$
$$\Rightarrow x < \frac{25}{6}$$

(i) It is evident that 1, 2, 3, and 4 are the only natural numbers less than $\frac{25}{6}$.

Thus, when x is a natural number, the solutions of the given inequality are 1, 2, 3, and 4.

Hence, in this case, the solution set is {1, 2, 3, 4}.

(ii) The integers less than $\frac{25}{6}$ are …−3, −2, −1, 0, 1, 2, 3, 4.

Thus, when x is an integer, the solutions of the given inequality are

…−3, −2, −1, 0, 1, 2, 3, 4.

Hence, in this case, the solution set is {…−3, −2, −1, 0, 1, 2, 3, 4}.

Question 2:

Solve −12x > 30, when

(i) x is a natural number (ii) x is an integer

Answer:

The given inequality is −12x > 30.

$$-12x > 30$$
$$\Rightarrow \frac{-12x}{-12} < \frac{30}{-12} \quad \left[\text{Dividing both sides by same negative number}\right]$$
$$\Rightarrow x < -\frac{5}{2}$$

(i) There is no natural number less than $\left(-\dfrac{5}{2}\right)$.

Thus, when x is a natural number, there is no solution of the given inequality.

(ii) The integers less than $\left(-\dfrac{5}{2}\right)$ are ..., −5, −4, −3.

Thus, when x is an integer, the solutions of the given inequality are

..., −5, −4, −3.

Hence, in this case, the solution set is {..., −5, −4, −3}.

Question 3:

Solve $5x - 3 < 7$, when

(i) x is an integer (ii) x is a real number

Answer:

The given inequality is $5x - 3 < 7$.

$$5x - 3 < 7$$
$$\Rightarrow 5x - 3 + 3 < 7 + 3$$
$$\Rightarrow 5x < 10$$
$$\Rightarrow \frac{5x}{5} < \frac{10}{5}$$
$$\Rightarrow x < 2$$

(i) The integers less than 2 are ..., −4, −3, −2, −1, 0, 1.

Thus, when x is an integer, the solutions of the given inequality are

..., −4, −3, −2, −1, 0, 1.

Hence, in this case, the solution set is {..., −4, −3, −2, −1, 0, 1}.

(ii) When x is a real number, the solutions of the given inequality are given by $x < 2$, that is, all real numbers x which are less than 2.

Thus, the solution set of the given inequality is $x \in (-\infty, 2)$.

Question 4:

Solve $3x + 8 > 2$, when

(i) x is an integer (ii) x is a real number

Answer:

The given inequality is $3x + 8 > 2$.

$$3x + 8 > 2$$
$$\Rightarrow 3x + 8 - 8 > 2 - 8$$
$$\Rightarrow 3x > -6$$
$$\Rightarrow \frac{3x}{3} > \frac{-6}{3}$$
$$\Rightarrow x > -2$$

(i) The integers greater than −2 are −1, 0, 1, 2, …

Thus, when x is an integer, the solutions of the given inequality are

−1, 0, 1, 2 …

Hence, in this case, the solution set is {−1, 0, 1, 2, …}.

(ii) When x is a real number, the solutions of the given inequality are all the real numbers, which are greater than −2.

Thus, in this case, the solution set is $(-2, \infty)$.

Question 5:

Solve the given inequality for real x: $4x + 3 < 5x + 7$

Answer:

$4x + 3 < 5x + 7$

$\Rightarrow 4x + 3 - 7 < 5x + 7 - 7$

$\Rightarrow 4x - 4 < 5x$

$\Rightarrow 4x - 4 - 4x < 5x - 4x$

⇒ −4 < x

Thus, all real numbers x, which are greater than −4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is (−4, ∞).

Question 6:

Solve the given inequality for real x: 3x − 7 > 5x − 1

Answer:

3x − 7 > 5x − 1

⇒ 3x − 7 + 7 > 5x − 1 + 7

⇒ 3x > 5x + 6

⇒ 3x − 5x > 5x + 6 − 5x

⇒ − 2x > 6

$$\Rightarrow \frac{-2x}{-2} < \frac{6}{-2}$$
$$\Rightarrow x < -3$$

Thus, all real numbers x, which are less than −3, are the solutions of the given inequality.

Hence, the solution set of the given inequality is (−∞, −3).

Question 7:

Solve the given inequality for real x: 3(x − 1) ≤ 2 (x − 3)

Answer:

3(x − 1) ≤ 2(x − 3)

⇒ 3x − 3 ≤ 2x − 6

⇒ 3x − 3 + 3 ≤ 2x − 6 + 3

⇒ 3x ≤ 2x − 3

⇒ 3x − 2x ≤ 2x − 3 − 2x

⇒ x ≤ − 3

Thus, all real numbers *x*, which are less than or equal to −3, are the solutions of the given inequality.

Hence, the solution set of the given inequality is (−∞, −3].

Question 8:

Solve the given inequality for real *x*: 3(2 − *x*) ≥ 2(1 − *x*)

Answer:

3(2 − *x*) ≥ 2(1 − *x*)

⇒ 6 − 3*x* ≥ 2 − 2*x*

⇒ 6 − 3*x* + 2*x* ≥ 2 − 2*x* + 2*x*

⇒ 6 − *x* ≥ 2

⇒ 6 − *x* − 6 ≥ 2 − 6

⇒ −*x* ≥ −4

⇒ *x* ≤ 4

Thus, all real numbers *x*, which are less than or equal to 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is (−∞, 4].

Question 9:

Solve the given inequality for real *x*: $x + \dfrac{x}{2} + \dfrac{x}{3} < 11$

Answer:

$$x + \frac{x}{2} + \frac{x}{3} < 11$$

$$\Rightarrow x\left(1 + \frac{1}{2} + \frac{1}{3}\right) < 11$$

$$\Rightarrow x\left(\frac{6+3+2}{6}\right) < 11$$

$$\Rightarrow \frac{11x}{6} < 11$$

$$\Rightarrow \frac{11x}{6 \times 11} < \frac{11}{11}$$

$$\Rightarrow \frac{x}{6} < 1$$

$$\Rightarrow x < 6$$

Thus, all real numbers x, which are less than 6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 6)$.

Question 10:

Solve the given inequality for real x: $\frac{x}{3} > \frac{x}{2} + 1$

Answer:

$$\frac{x}{3} > \frac{x}{2} + 1$$

$$\Rightarrow \frac{x}{3} - \frac{x}{2} > 1$$

$$\Rightarrow \frac{2x - 3x}{6} > 1$$

$$\Rightarrow -\frac{x}{6} > 1$$

$$\Rightarrow -x > 6$$

$$\Rightarrow x < -6$$

Thus, all real numbers x, which are less than −6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, -6)$.

Question 11:

Solve the given inequality for real x: $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

Answer:

$$\frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$$

$$\Rightarrow 9(x-2) \le 25(2-x)$$

$$\Rightarrow 9x - 18 \le 50 - 25x$$

$$\Rightarrow 9x - 18 + 25x \le 50$$

$$\Rightarrow 34x - 18 \le 50$$

$$\Rightarrow 34x \le 50 + 18$$

$$\Rightarrow 34x \le 68$$

$$\Rightarrow \frac{34x}{34} \le \frac{68}{34}$$

$$\Rightarrow x \le 2$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

Question 12:

Solve the given inequality for real x: $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$

Answer:

$$\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$$

$$\Rightarrow 3\left(\frac{3x}{5}+4\right) \ge 2(x-6)$$

$$\Rightarrow \frac{9x}{5} + 12 \ge 2x - 12$$

$$\Rightarrow 12 + 12 \ge 2x - \frac{9x}{5}$$

$$\Rightarrow 24 \ge \frac{10x - 9x}{5}$$

$$\Rightarrow 24 \ge \frac{x}{5}$$

$$\Rightarrow 120 \ge x$$

Thus, all real numbers x, which are less than or equal to 120, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 120]$.

Question 13:

Solve the given inequality for real x: $2(2x + 3) - 10 < 6(x - 2)$

Answer:

$$2(2x+3)-10 < 6(x-2)$$
$$\Rightarrow 4x+6-10 < 6x-12$$
$$\Rightarrow 4x-4 < 6x-12$$
$$\Rightarrow -4+12 < 6x-4x$$
$$\Rightarrow 8 < 2x$$
$$\Rightarrow 4 < x$$

Thus, all real numbers x, which are greater than or equal to 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4, \infty)$.

Question 14:

Solve the given inequality for real x: $37 - (3x + 5) \geq 9x - 8(x - 3)$

Answer:

$$37-(3x+5) \geq 9x-8(x-3)$$
$$\Rightarrow 37-3x-5 \geq 9x-8x+24$$
$$\Rightarrow 32-3x \geq x+24$$
$$\Rightarrow 32-24 \geq x+3x$$
$$\Rightarrow 8 \geq 4x$$
$$\Rightarrow 2 \geq x$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

Question 15:

Solve the given inequality for real x: $\dfrac{x}{4} < \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

Answer:

$$\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

$$\Rightarrow \frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$$

$$\Rightarrow \frac{x}{4} < \frac{25x-10-21x+9}{15}$$

$$\Rightarrow \frac{x}{4} < \frac{4x-1}{15}$$

$$\Rightarrow 15x < 4(4x-1)$$

$$\Rightarrow 15x < 16x-4$$

$$\Rightarrow 4 < 16x-15x$$

$$\Rightarrow 4 < x$$

Thus, all real numbers x, which are greater than 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4, \infty)$.

Question 16:

Solve the given inequality for real x: $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Answer:

$$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$$

$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$$

$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{15x-10-8+4x}{20}$$

$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{19x-18}{20}$$

$$\Rightarrow 20(2x-1) \geq 3(19x-18)$$

$$\Rightarrow 40x-20 \geq 57x-54$$

$$\Rightarrow -20+54 \geq 57x-40x$$

$$\Rightarrow 34 \geq 17x$$

$$\Rightarrow 2 \geq x$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

Question 17:

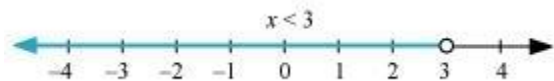Solve the given inequality and show the graph of the solution on number line: $3x - 2 < 2x + 1$

Answer:

$3x - 2 < 2x + 1$

$\Rightarrow 3x - 2x < 1 + 2$

$\Rightarrow x < 3$

The graphical representation of the solutions of the given inequality is as follows.

Question 18:

Solve the given inequality and show the graph of the solution on number line: $5x - 3 \geq 3x - 5$

Answer:

$$5x - 3 \geq 3x - 5$$
$$\Rightarrow 5x - 3x \geq -5 + 3$$
$$\Rightarrow 2x \geq -2$$
$$\Rightarrow \frac{2x}{2} \geq \frac{-2}{2}$$
$$\Rightarrow x \geq -1$$

The graphical representation of the solutions of the given inequality is as follows.

Question 19:

Solve the given inequality and show the graph of the solution on number line: $3(1 - x) < 2(x + 4)$

Answer:

$$3(1-x) < 2(x+4)$$
$$\Rightarrow 3-3x < 2x+8$$
$$\Rightarrow 3-8 < 2x+3x$$
$$\Rightarrow -5 < 5x$$
$$\Rightarrow \frac{-5}{5} < \frac{5x}{5}$$
$$\Rightarrow -1 < x$$

The graphical representation of the solutions of the given inequality is as follows.

Question 20:

Solve the given inequality and show the graph of the solution on number

line: $\dfrac{x}{2} \geq \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

Answer:

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$
$$\Rightarrow \frac{x}{2} \geq \frac{5(5x-2) - 3(7x-3)}{15}$$
$$\Rightarrow \frac{x}{2} \geq \frac{25x - 10 - 21x + 9}{15}$$
$$\Rightarrow \frac{x}{2} \geq \frac{4x-1}{15}$$
$$\Rightarrow 15x \geq 2(4x-1)$$
$$\Rightarrow 15x \geq 8x - 2$$
$$\Rightarrow 15x - 8x \geq 8x - 2 - 8x$$
$$\Rightarrow 7x \geq -2$$
$$\Rightarrow x \geq -\frac{2}{7}$$

The graphical representation of the solutions of the given inequality is as follows.
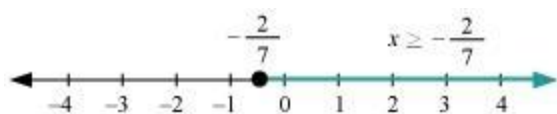
Question 21:

Ravi obtained 70 and 75 marks in first two unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Answer:

Let x be the marks obtained by Ravi in the third unit test.

Since the student should have an average of at least 60 marks,

$$\frac{70+75+x}{3} \geq 60$$
$$\Rightarrow 145 + x \geq 180$$
$$\Rightarrow x \geq 180 - 145$$
$$\Rightarrow x \geq 35$$

Thus, the student must obtain a minimum of 35 marks to have an average of at least 60 marks.

Question 22:

To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

Answer:

Let x be the marks obtained by Sunita in the fifth examination.

In order to receive grade 'A' in the course, she must obtain an average of 90 marks or more in five examinations.

Therefore,

$$\frac{87+92+94+95+x}{5} \geq 90$$
$$\Rightarrow \frac{368+x}{5} \geq 90$$
$$\Rightarrow 368 + x \geq 450$$
$$\Rightarrow x \geq 450 - 368$$
$$\Rightarrow x \geq 82$$

Thus, Sunita must obtain greater than or equal to 82 marks in the fifth examination.

Question 23:

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Answer:

Let *x* be the smaller of the two consecutive odd positive integers. Then, the other integer is *x* + 2.

Since both the integers are smaller than 10,

x + 2 < 10

⇒ *x* < 10 − 2

⇒ *x* < 8 … (i)

Also, the sum of the two integers is more than 11.

∴*x* + (*x* + 2) > 11

⇒ 2*x* + 2 > 11

⇒ 2*x* > 11 − 2

⇒ 2*x* > 9

$$\Rightarrow x > \frac{9}{2}$$
$$\Rightarrow x > 4.5 \qquad \qquad \ldots(ii)$$

From (i) and (ii), we obtain

$$\Rightarrow x > \frac{9}{2}$$
$$\Rightarrow x > 4.5 \qquad \qquad \ldots(ii)$$

$$\Rightarrow x < \frac{21}{2}$$
$$\Rightarrow x < 10.5 \qquad \qquad \ldots(2)$$

$$x \text{ cm} + 3x \text{ cm} + (3x - 2) \text{ cm} \geq 61 \text{ cm}$$
$$\Rightarrow 7x - 2 \geq 61$$
$$\Rightarrow 7x \geq 61 + 2$$
$$\Rightarrow 7x \geq 63$$
$$\Rightarrow \frac{7x}{7} \geq \frac{63}{7}$$
$$\Rightarrow x \geq 9$$

$$\Rightarrow \frac{4x}{4} \leq \frac{88}{4}$$
$$\Rightarrow x \leq 22 \qquad \qquad \dots(1)$$

Question 24 :

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Answer :

Let x be the smaller of the two consecutive even positive integers .

Then the other integer is x+2.

Since both the integers are larger than 5,x>5 ....(1)

Also the sum of the two integers is less than 23.

x+(x+2)<23

⇒2x+2<23

⇒2x<23−2

⇒2x<21

⇒x<

2

21

⇒x<10.5....(2)

From (1) and (2) we obtain 5<x<10.5.

Since x is an even number, x can take the values 6,8 and 10.

Thus the required possible pairs are (6,8),(8,10) and (10,12).

Question 25:

The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length in cm. of the shortest side.

 Answer :

Correct option is A)
Let the length of the shortest side of the triangle be x cm.
Then length of the longest side =3x cm.
Thus the length of the third side =(3x−2) cm.
Since the perimeter of the triangle is at least 61 cm,
x+3x+(3x−2) ≥ 61
⇒7x−2≥61
⇒7x≥61+2
⇒7x≥63
⇒x≥9
Thus the minimum length of the shortest side is 9 cm.

Question 26:

A man wants to cut three lengths from a single piece of board of length 91cm.The second

length is to be 3cm longer than the shortest and the third length is to be twice as long as the

shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5

cm longer than the second?

Answer :

Let the length of the shortest piece be x cm .
Then the length of the second piece and the third piece are (x+3) cm and 2x cm respectively.
Since the three lengths are to be cut from a single piece of board of length 91 cm,
x+(x+3)+2x ≤ 91
⇒4x+3≤91
⇒4x≤91−3
⇒4x≤88
⇒
4
4x

≤
4
88

⇒x≤22-----(i)
Also the third piece is at least 5 cm longer than the second piece.
∴2x≥(x+3)+5
⇒2x≥x+8
⇒x≥8....(2)
From (1) and (2) we obtain
8≤x≤22
Thus the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.

Miscellaneous Exercise

Question 1:

Solve the inequality $2 \leq 3x - 4 \leq 5$

Answer:

$2 \leq 3x - 4 \leq 5$

$\Rightarrow 2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$

$\Rightarrow 6 \leq 3x \leq 9$

$\Rightarrow 2 \leq x \leq 3$

Thus, all the real numbers, x, which are greater than or equal to 2 but less than or equal to 3, are the solutions of the given inequality. The solution set for the given inequality is [2, 3].

Question 2:

Solve the inequality $6 \leq -3(2x - 4) < 12$

Answer:

$6 \leq -3(2x - 4) < 12$

$\Rightarrow 2 \leq -(2x - 4) < 4$

$\Rightarrow -2 \geq 2x - 4 > -4$

$\Rightarrow 4 - 2 \geq 2x > 4 - 4$

$\Rightarrow 2 \geq 2x > 0$

$\Rightarrow 1 \geq x > 0$

Thus, the solution set for the given inequality is (0, 1].

Question 3:

Solve the inequality $-3 \le 4 - \dfrac{7x}{2} \le 18$

Answer:

$$-3 \le 4 - \frac{7x}{2} \le 18$$

$$\Rightarrow -3 - 4 \le -\frac{7x}{2} \le 18 - 4$$

$$\Rightarrow -7 \le -\frac{7x}{2} \le 14$$

$$\Rightarrow 7 \ge \frac{7x}{2} \ge -14$$

$$\Rightarrow 1 \ge \frac{x}{2} \ge -2$$

$$\Rightarrow 2 \ge x \ge -4$$

Thus, the solution set for the given inequality is [−4, 2].

Question 4:

Solve the inequality $-15 < \dfrac{3(x-2)}{5} \le 0$

Answer:

$$-15 < \frac{3(x-2)}{5} \le 0$$

$$\Rightarrow -75 < 3(x-2) \le 0$$

$$\Rightarrow -25 < x - 2 \le 0$$

$$\Rightarrow -25 + 2 < x \le 2$$

$$\Rightarrow -23 < x \le 2$$

Thus, the solution set for the given inequality is (−23, 2].

Question 5:

Solve the inequality $-12 < 4 - \dfrac{3x}{-5} \le 2$

Answer:

$$-12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 - 4 < \frac{-3x}{-5} \leq 2 - 4$$

$$\Rightarrow -16 < \frac{3x}{5} \leq -2$$

$$\Rightarrow -80 < 3x \leq -10$$

$$\Rightarrow \frac{-80}{3} < x \leq \frac{-10}{3}$$

Thus, the solution set for the given inequality is $\left(\frac{-80}{3}, \frac{-10}{3} \right]$.

Question 6:

Solve the inequality $7 \leq \frac{(3x+11)}{2} \leq 11$

Answer:

$$7 \leq \frac{(3x+11)}{2} \leq 11$$

$$\Rightarrow 14 \leq 3x + 11 \leq 22$$

$$\Rightarrow 14 - 11 \leq 3x \leq 22 - 11$$

$$\Rightarrow 3 \leq 3x \leq 11$$

$$\Rightarrow 1 \leq x \leq \frac{11}{3}$$

Thus, the solution set for the given inequality is $\left[1, \frac{11}{3} \right]$.

Question 7:

Solve the inequalities and represent the solution graphically on number line: $5x + 1 > -24$, $5x - 1 < 24$
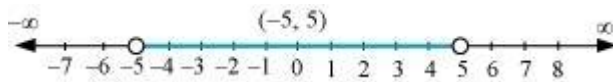
Answer:

$5x + 1 > -24$

$\Rightarrow 5x > -25$

$\Rightarrow x > -5 \dots (1)$

$5x - 1 < 24$

$\Rightarrow 5x < 25$

$\Rightarrow x < 5 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(-5, 5)$. The solution of the given system of inequalities can be represented on number line as

Question 8:

Solve the inequalities and represent the solution graphically on number line: $2(x - 1) < x + 5, 3(x + 2) > 2 - x$

Answer:

$2(x - 1) < x + 5$

$\Rightarrow 2x - 2 < x + 5$

$\Rightarrow 2x - x < 5 + 2$
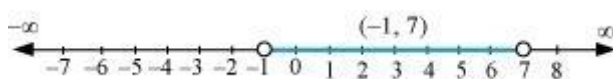
$\Rightarrow x < 7 \ldots (1)$

$3(x + 2) > 2 - x$

$\Rightarrow 3x + 6 > 2 - x$

$\Rightarrow 3x + x > 2 - 6$

$\Rightarrow 4x > -4$

$\Rightarrow x > -1 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(-1, 7)$. The solution of the given system of inequalities can be represented on number line as

Question 9:

Solve the following inequalities and represent the solution graphically on number line:

$3x - 7 > 2(x - 6), 6 - x > 11 - 2x$

Answer:

$3x - 7 > 2(x - 6)$

$\Rightarrow 3x - 7 > 2x - 12$

$\Rightarrow 3x - 2x > -12 + 7$
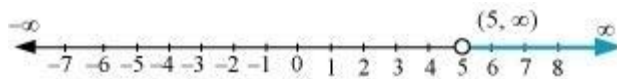
$\Rightarrow x > -5 \ldots (1)$

$6 - x > 11 - 2x$

$\Rightarrow -x + 2x > 11 - 6$

$\Rightarrow x > 5 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(5, \infty)$. The solution of the given system of inequalities can be represented on number line as

Question 10:

Solve the inequalities and represent the solution graphically on number line: $5(2x - 7) - 3(2x + 3) \leq 0$, $2x + 19 \leq 6x + 47$

Answer:

$5(2x - 7) - 3(2x + 3) \leq 0$

$\Rightarrow 10x - 35 - 6x - 9 \leq 0$

$\Rightarrow 4x - 44 \leq 0$

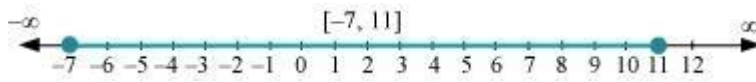$\Rightarrow 4x \leq 44$

$\Rightarrow x \leq 11 \ldots (1)$

$2x + 19 \leq 6x + 47$

$\Rightarrow 19 - 47 \leq 6x - 2x$

$\Rightarrow -28 \leq 4x$

$\Rightarrow -7 \leq x \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is [−7, 11]. The solution of the given system of inequalities can be represented on number line as

Question 11:

A solution is to be kept between 68°F and 77°F. What is the range in temperature in degree Celsius (C) if the Celsius/Fahrenheit (F) conversion formula is given by $F = \dfrac{9}{8}C + 32$?

Answer:

Since the solution is to be kept between 68°F and 77°F,

68 < F < 77

Putting $F = \dfrac{9}{5}C + 32,$ we obtain

$$68 < \dfrac{9}{5}C + 32 < 77$$

$$\Rightarrow 68 - 32 < \dfrac{9}{5}C < 77 - 32$$

$$\Rightarrow 36 < \dfrac{9}{5}C < 45$$

$$\Rightarrow 36 \times \dfrac{5}{9} < C < 45 \times \dfrac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

Thus, the required range of temperature in degree Celsius is between 20°C and 25°C.

Question 12:

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Answer:

Let *x* litres of 2% boric acid solution is required to be added.

Then, total mixture = (*x* + 640) litres

This resulting mixture is to be more than 4% but less than 6% boric acid.

∴2%x + 8% of 640 > 4% of (x + 640)

And, 2% x + 8% of 640 < 6% of (x + 640)

2%x + 8% of 640 > 4% of (x + 640)

$$\Rightarrow \frac{2}{100}x + \frac{8}{100}(640) > \frac{4}{100}(x+640)$$

⇒ 2x + 5120 > 4x + 2560

⇒ 5120 − 2560 > 4x − 2x

⇒ 5120 − 2560 > 2x

⇒ 2560 > 2x

⇒ 1280 > x

2% x + 8% of 640 < 6% of (x + 640)

$$\frac{2}{100}x + \frac{8}{100}(640) < \frac{6}{100}(x+640)$$

⇒ 2x + 5120 < 6x + 3840

⇒ 5120 − 3840 < 6x − 2x

⇒ 1280 < 4x

⇒ 320 < x

∴320 < x < 1280

Thus, the number of litres of 2% of boric acid solution that is to be added will have to be more than 320 litres but less than 1280 litres.

Question 13:

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Answer:

Let x litres of water is required to be added.

Then, total mixture = (x + 1125) litres

It is evident that the amount of acid contained in the resulting mixture is 45% of 1125 litres.

This resulting mixture will contain more than 25% but less than 30% acid content.

∴30% of (1125 + x) > 45% of 1125

And, 25% of (1125 + x) < 45% of 1125

30% of (1125 + x) > 45% of 1125

$$\Rightarrow \frac{30}{100}(1125 + x) > \frac{45}{100} \times 1125$$
$$\Rightarrow 30(1125 + x) > 45 \times 1125$$
$$\Rightarrow 30 \times 1125 + 30x > 45 \times 1125$$
$$\Rightarrow 30x > 45 \times 1125 - 30 \times 1125$$
$$\Rightarrow 30x > (45 - 30) \times 1125$$
$$\Rightarrow x > \frac{15 \times 1125}{30} = 562.5$$

25% of (1125 + x) < 45% of 1125

⇒251001125 + x<45100×1125 ⇒251125 + x<45×1125

⇒25×1125 +25x<45×1125 ⇒25x<45×1125 -25×1125

⇒25x<22500⇒x<900

∴ 562.5 < x < 900

Thus, the required number of litres of water that is to be added will have to be more than 562.5 but less than 900.

Question 14:

IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100,$$

Where MA is mental age and CA is chronological age. If 80 ≤ IQ ≤ 140 for a group of 12 years old children, find the range of their mental age.

Answer:

It is given that for a group of 12 years old children, 80 ≤ IQ ≤ 140 … (i)

For a group of 12 years old children, CA = 12 years

$$IQ = \frac{MA}{12} \times 100$$

Putting this value of IQ in (i), we obtain

$$80 \leq \frac{MA}{12} \times 100 \leq 140$$

$$\Rightarrow 80 \times \frac{12}{100} \leq MA \leq 140 \times \frac{12}{100}$$

$$\Rightarrow 9.6 \leq MA \leq 16.8$$

Thus, the range of mental age of the group of 12 years old children is $9.6 \leq MA \leq 16.8$.